Model Selection and Comparison

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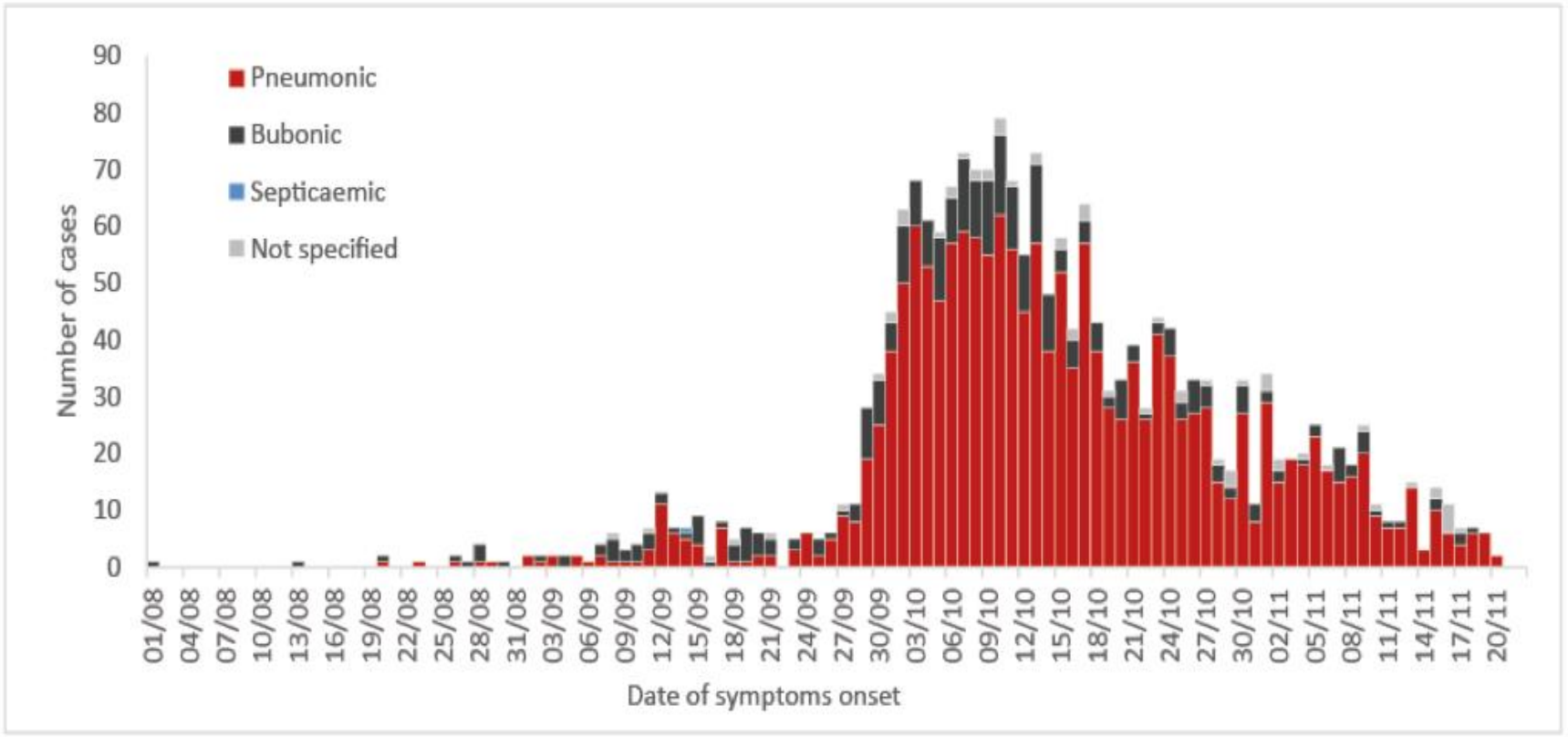
E2M2 2022 Ranomafana, Madagascar

Which model is best?



http://www.who.int/csr/don/27-november-2017-plague-madagascar/en/

There are many statistical methods used to ‘fit’ models to data and there are many scenarios from which mechanical model can be built.

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The method best suited for your work will depend on your model and your data.

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What are some measures of model fit used in E2M2 so far?

R squared R-carré

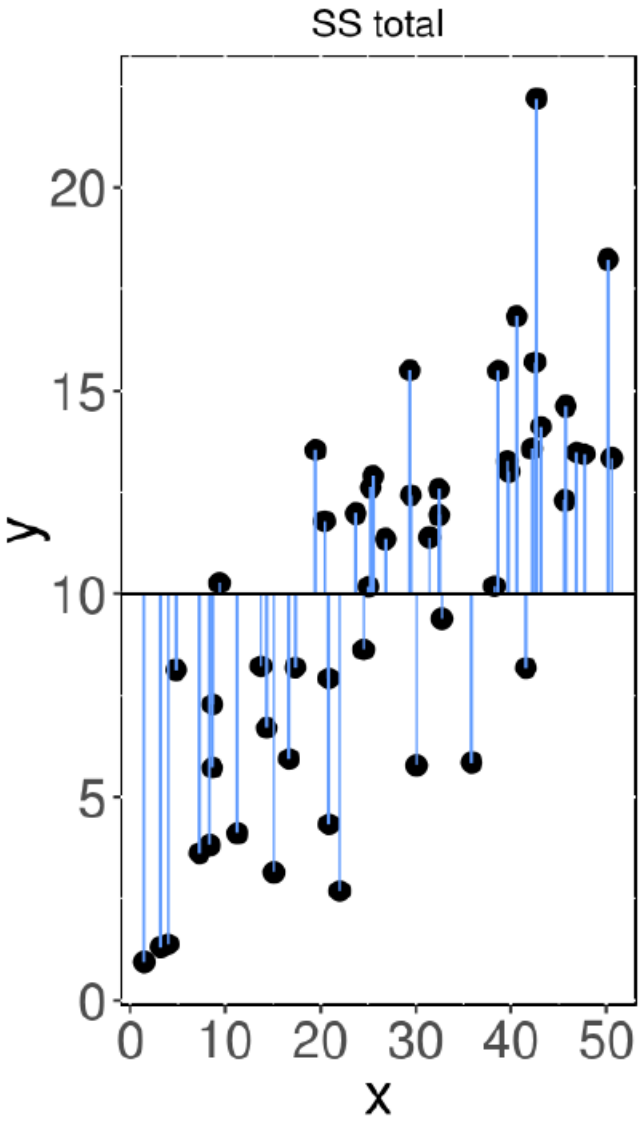
Least squares (Moindres carrés)

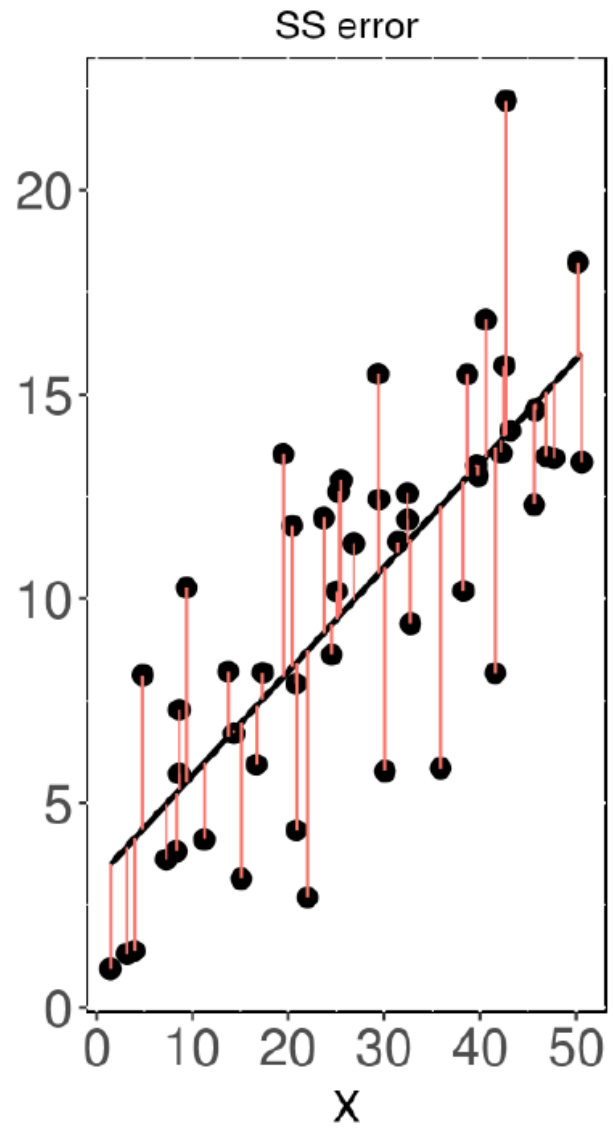
Log likelihood Maximum de vraisemblance

AIC

(uses least squares or log-likelihood but penalizes by number of fitted parameters)

Definition r^2



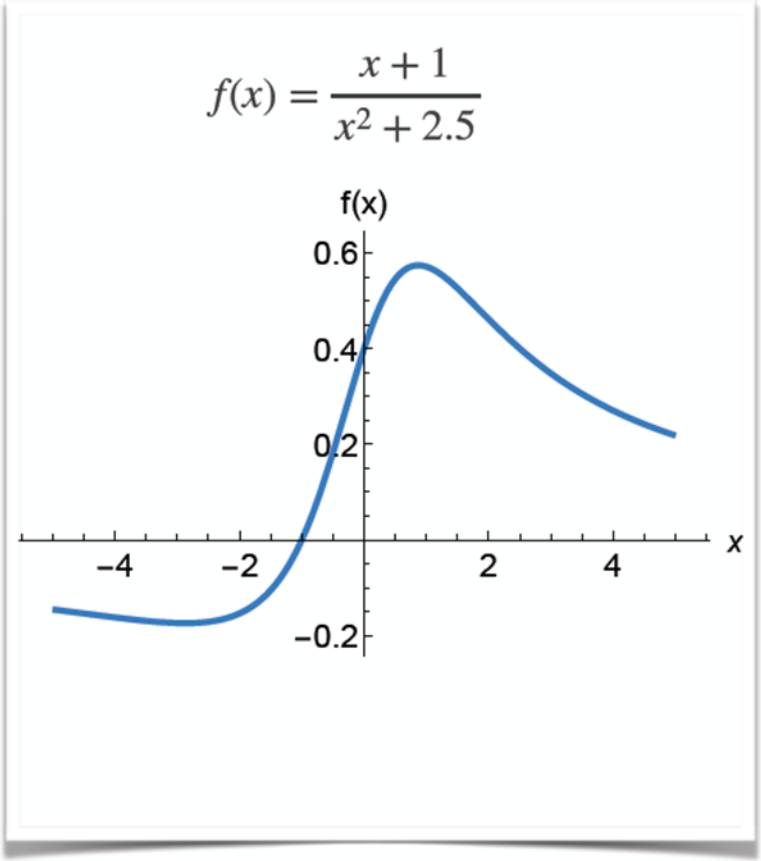


$$R^2 = 1 - \frac{SSE_p}{SST}$$

Optimization/maximization

Function properties

?

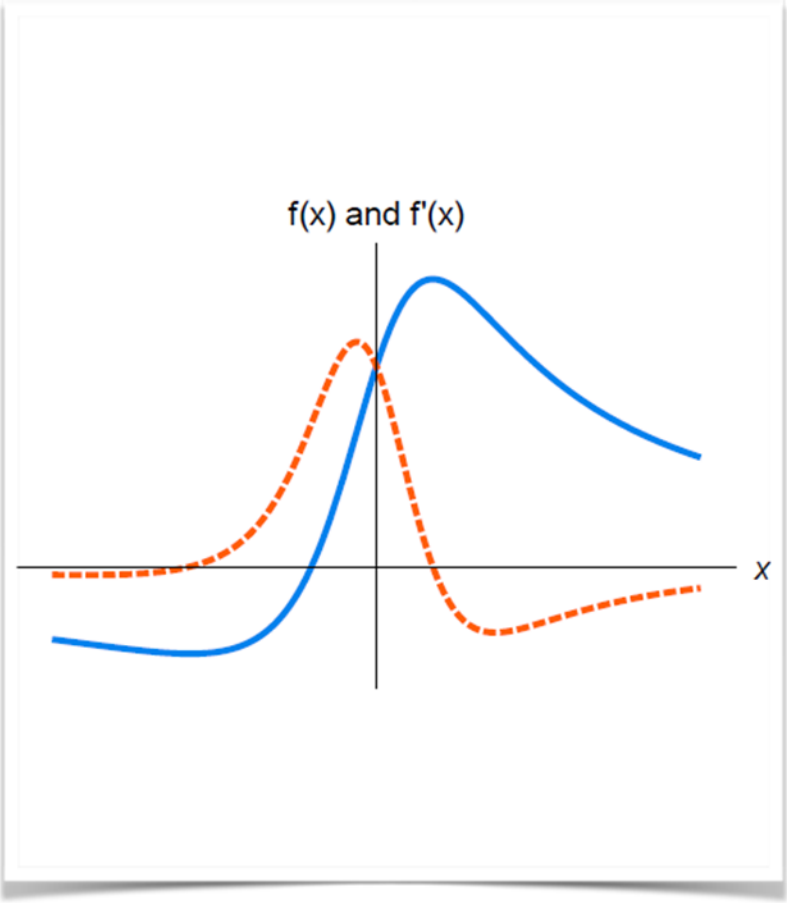


From Tanjona Ramiadantsoa

Optimization/maximization

A function and its derivative

- What happen when the derivative is:
  - negative?
  - positive?
  - zero?
  - reaching a maximum (finite) value?



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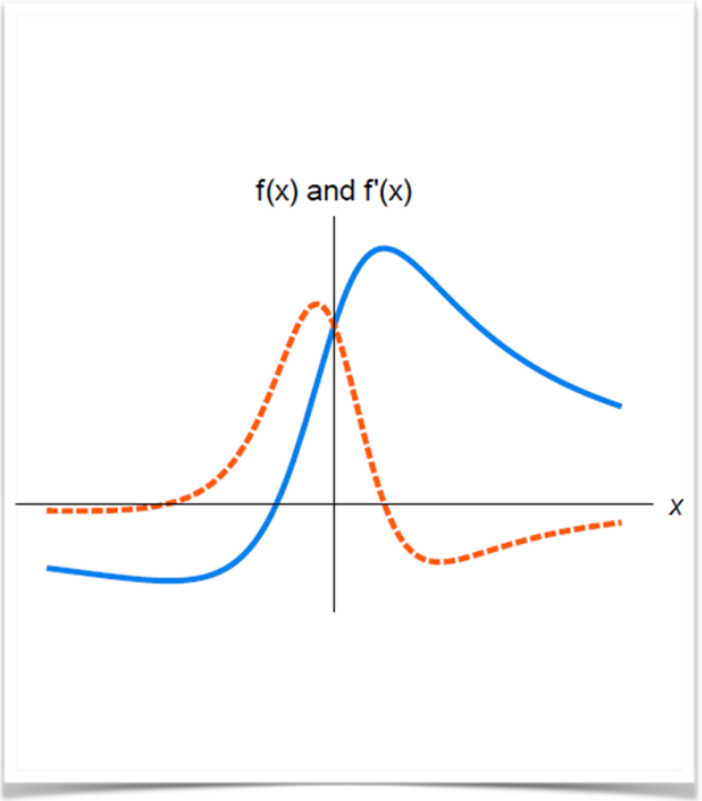
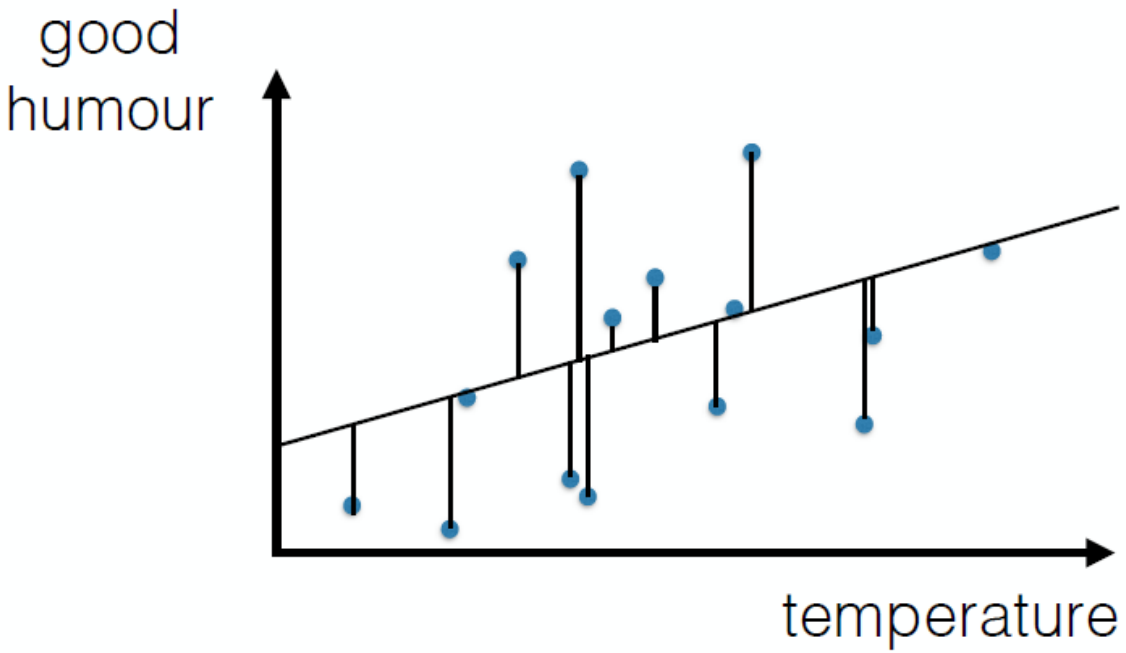


From Tanjona Ramiadantsoa

The R function ‘optim’ can be used to minimize these measures of model difference from the data.

Least squares

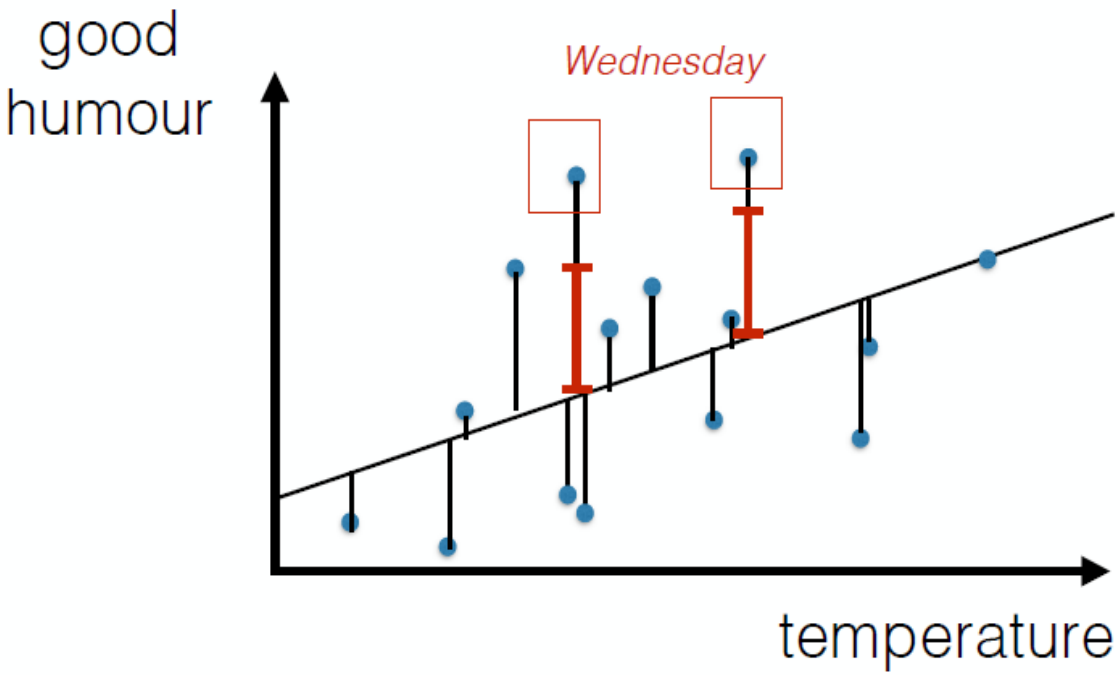
Adding covariates and R^2



$$\text{humour} = b_0 + b_1\text{temperature} + \text{Error}$$

Least squares

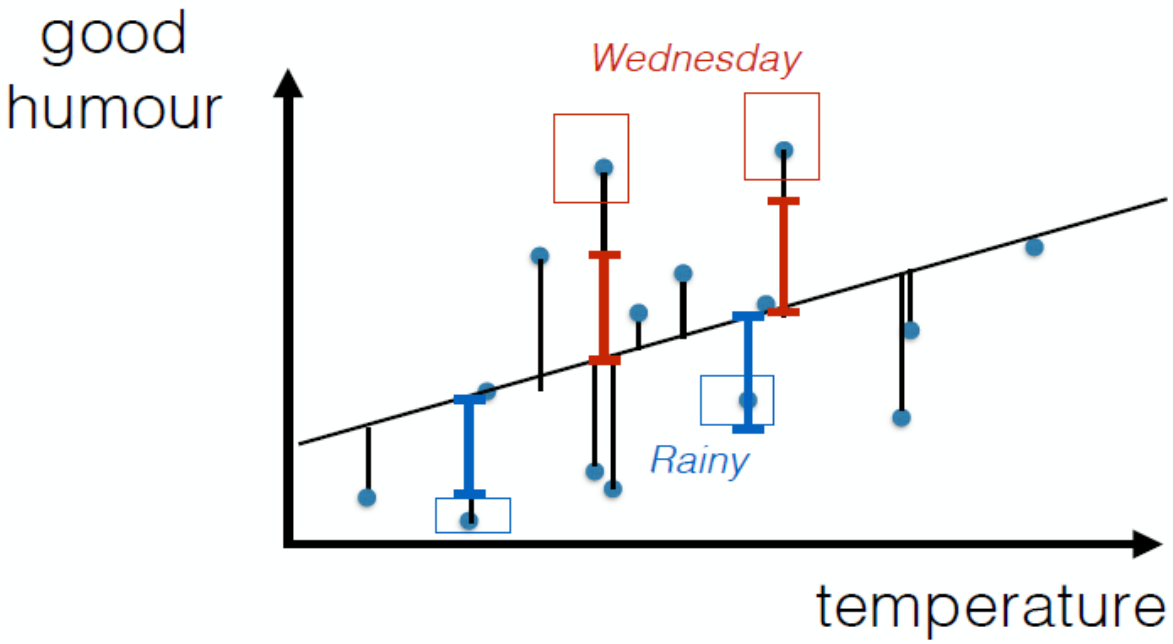
Adding covariates and R^2



$$humour = b_0 + b_1 temperature + b_2 Wednesday + Error$$

Least squares

Adding covariates and R^2



$$humour = b_0 + b_1 temperature + b_2 Wednesday + b_3 rain + Error$$

Least squares

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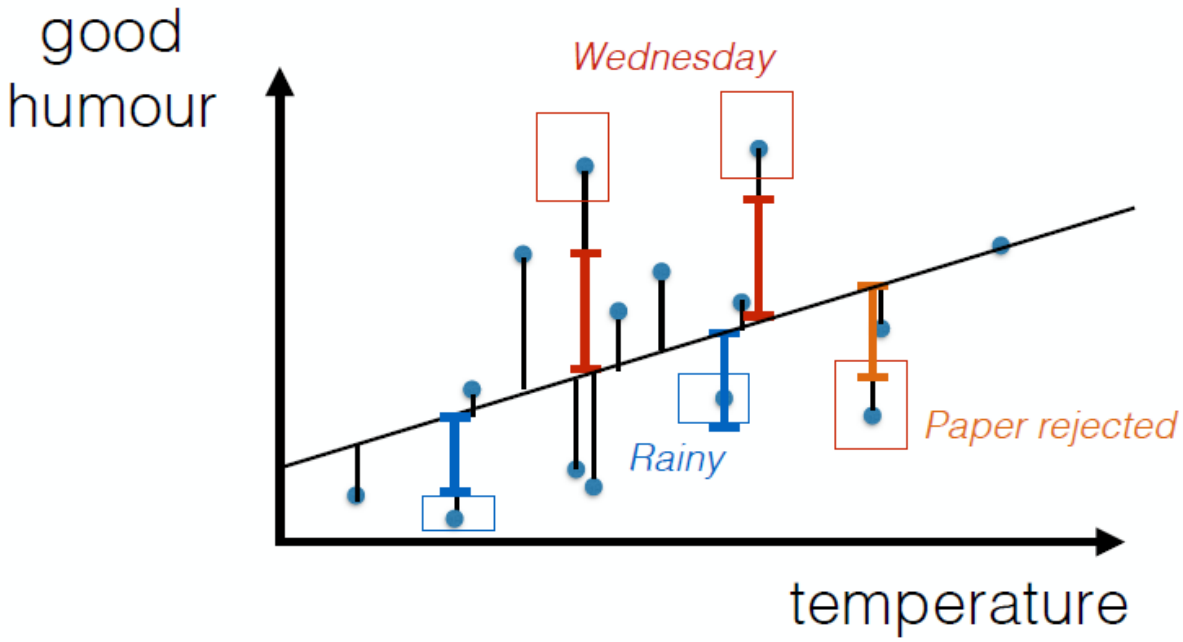
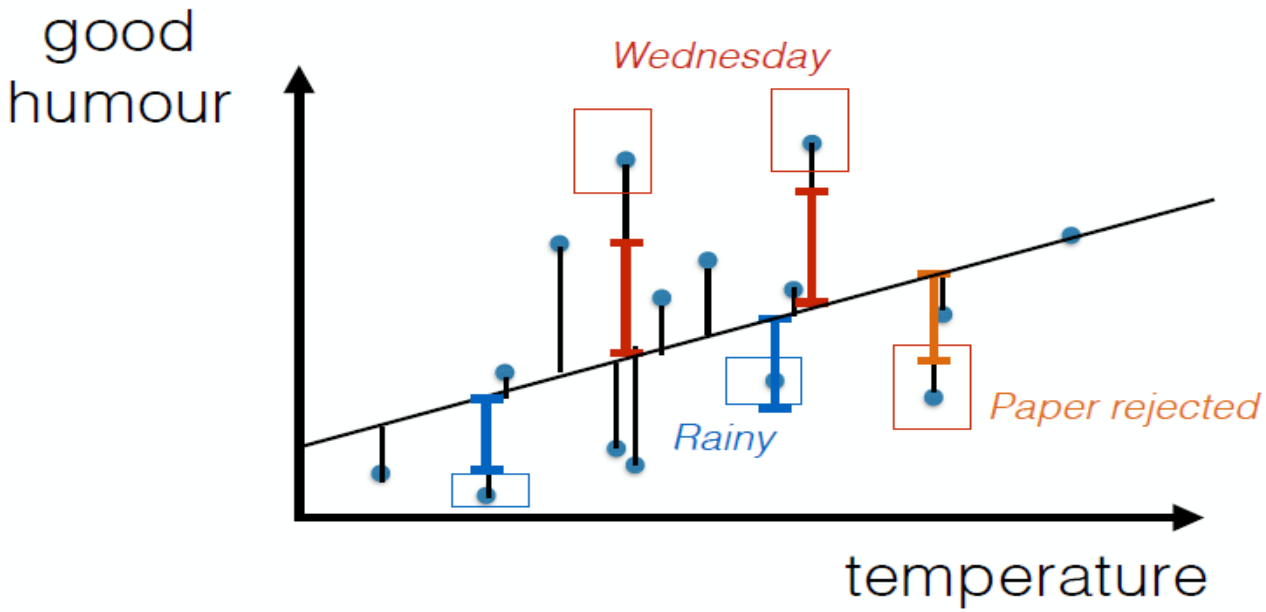


$$\text{humour} = b_0 + b_1\text{temperature} + b_2\text{Wednesday} + b_3\text{rain} + b_4\text{rejection} + \text{Error}$$

Least squares

Adding covariates and R^2



Adding covariates almost always increases the R^2 - so a key question is when to stop.

What to choose?

Least square AIC

$$\mathrm{AIC} = N * ln(\frac{SS_e}{N}) + 2K$$

$\boldsymbol{N}$: *Number of observations*
$\boldsymbol{SS_e}$: *Sum square of errors*
$\boldsymbol{K}$: *Number of parameters*

The smaller the AIC the better

Least square AIC

More parameter is not always good

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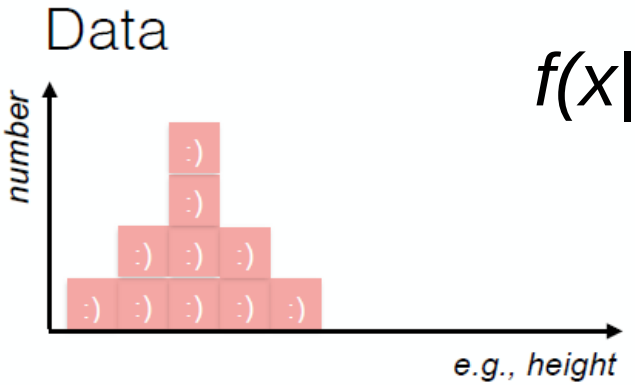
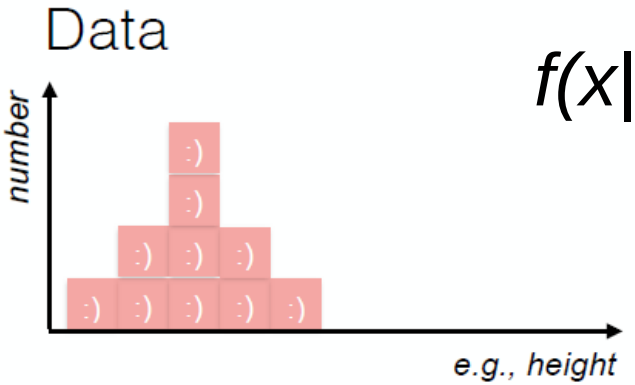
Maximum likelihood



$f(x|\mu, \sigma^2)$

Maximum likelihood



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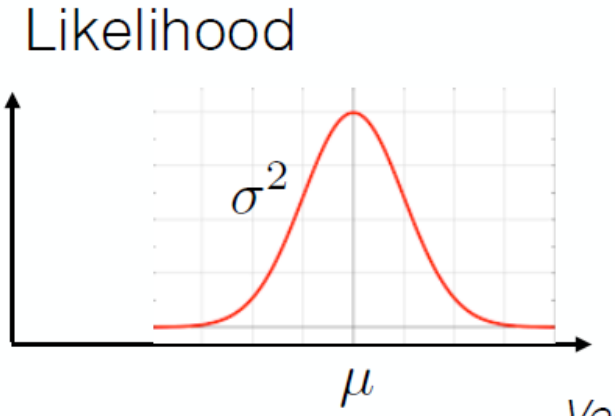
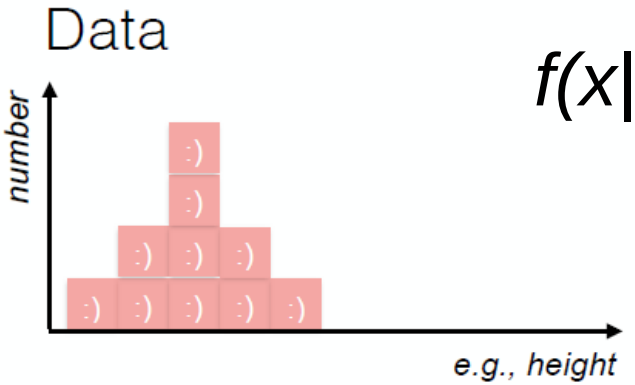


~ the 'probability of seeing the data' given the chosen parameters
…each individual in the data has a corresponding likelihood… multiply them…

Very low likelihood.

Maximum likelihood



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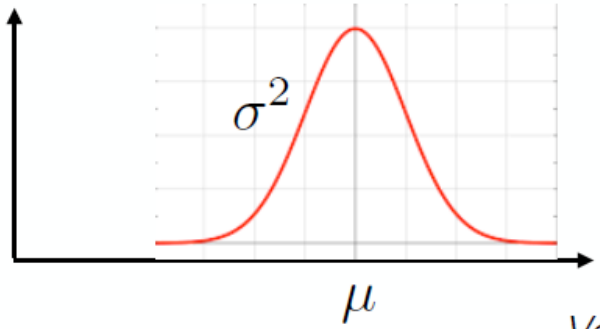
Likelihood



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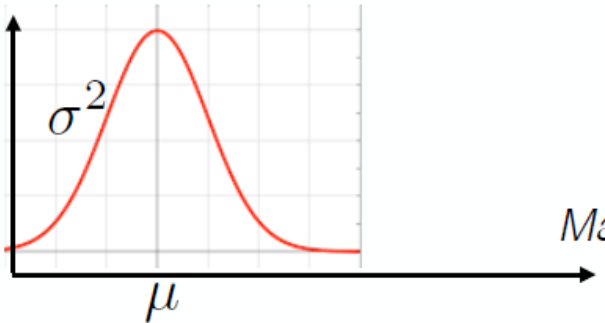

Maximum likelihood

An example of model selection: *Bartonella spp.* in Madagascar rats

Epidemics 20 (2017) 56–66

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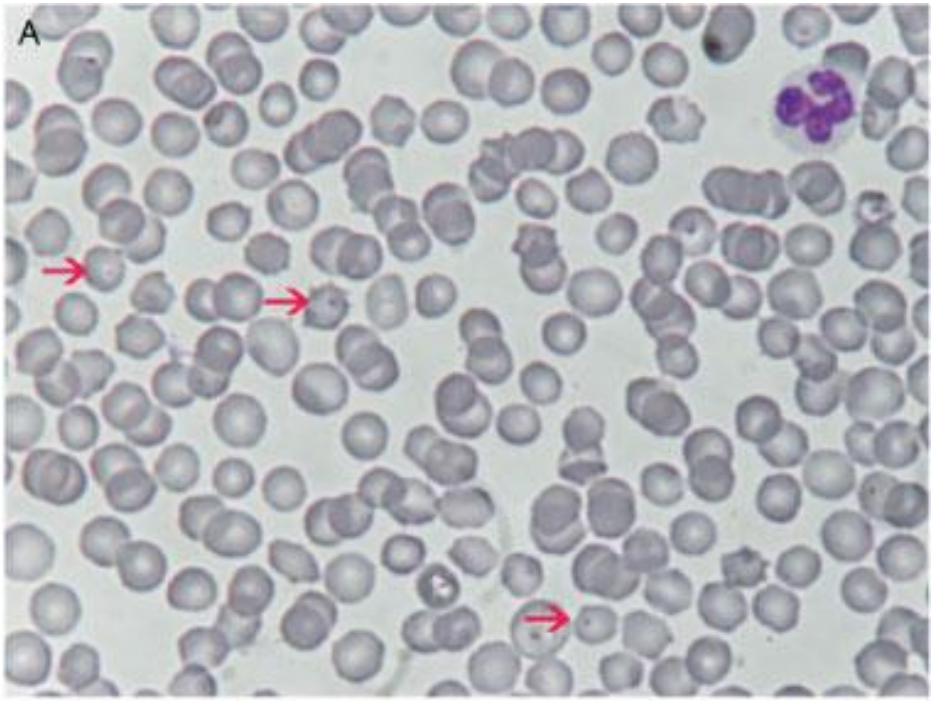
journal homepage: www.elsevier.com/locate/epidemics

Elucidating transmission dynamics and host-parasite-vector relationships for rodent-borne *Bartonella* spp. in Madagascar

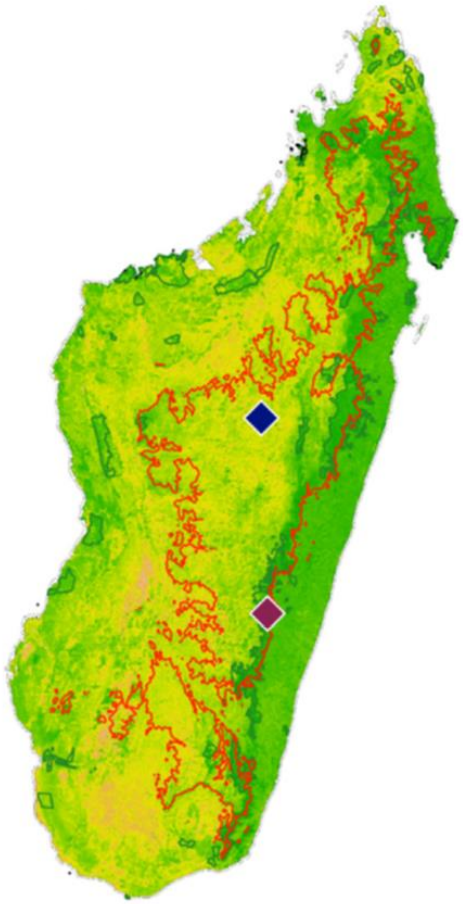
Cara E. Brook[a,*], Ying Bai[b], Emily O. Yu[a], Hafaliana C. Ranaivoson[c,d], Haewon Shin[e], Andrew P. Dobson[a], C. Jessica E. Metcalf[a,1], Michael Y. Kosoy[b,1], Katharina Dittmar[e,1]

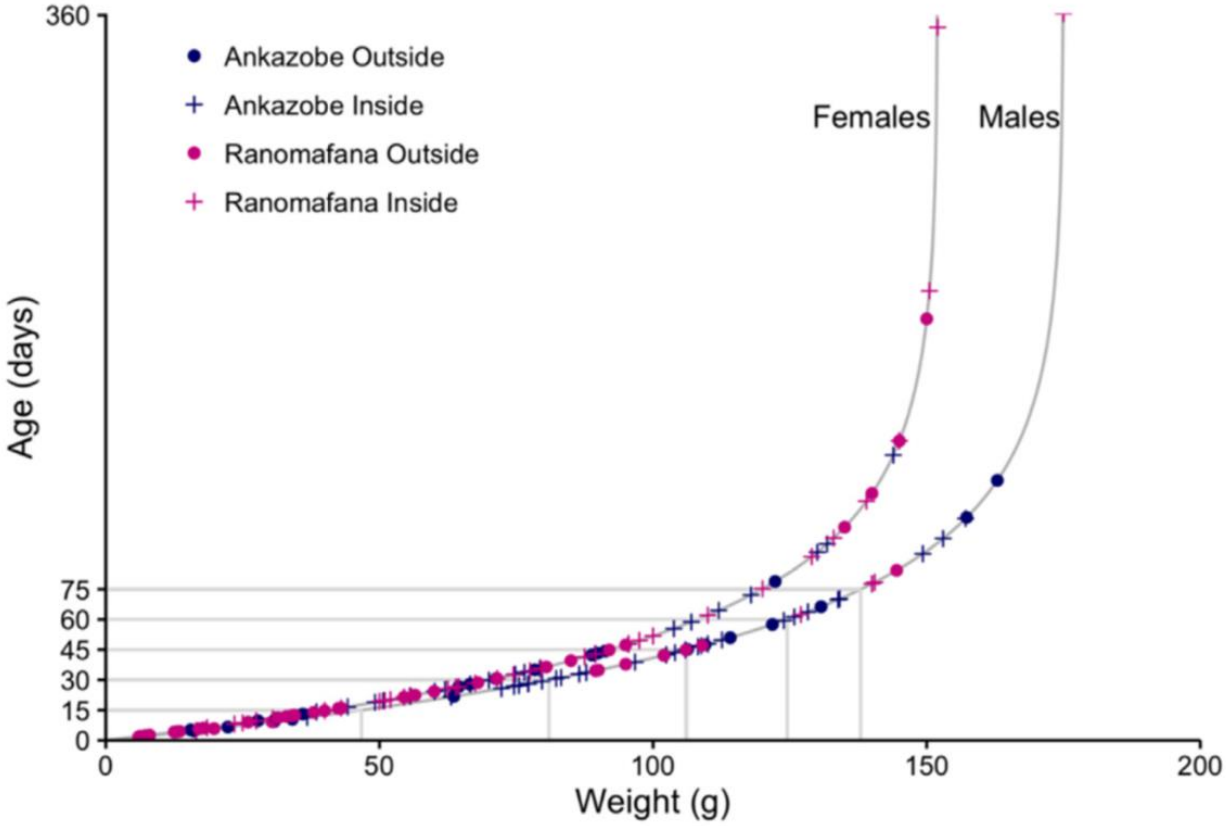
Bartonella spp.

- persistent erythrocyctic bacteria that are sometimes zoonotic
- vectored by ticks, fleas, sand flies, mosquitoes
- at least 8 human-infecting species
  - *Bartonella bacilliformis* = Carrion's disease
  - *Bartonella henselae* = cat scratch fever
  - *Bartonella quintana* = trench fever

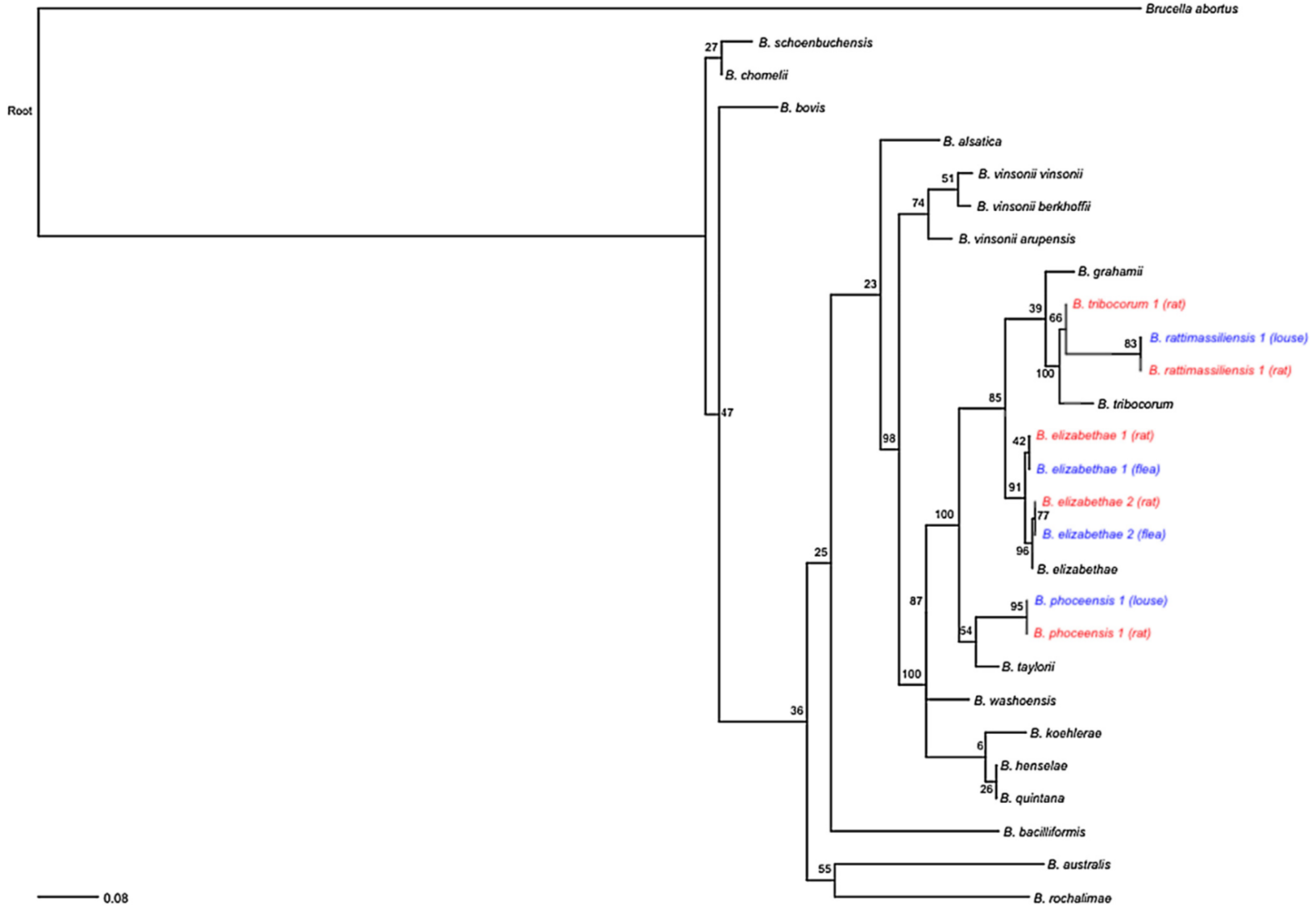


We first collected samples from rats in two sites Madagascar.





Statistically, we demonstrated an association between genotypes of *Bartonella* spp. found in rats and their ectoparasites.



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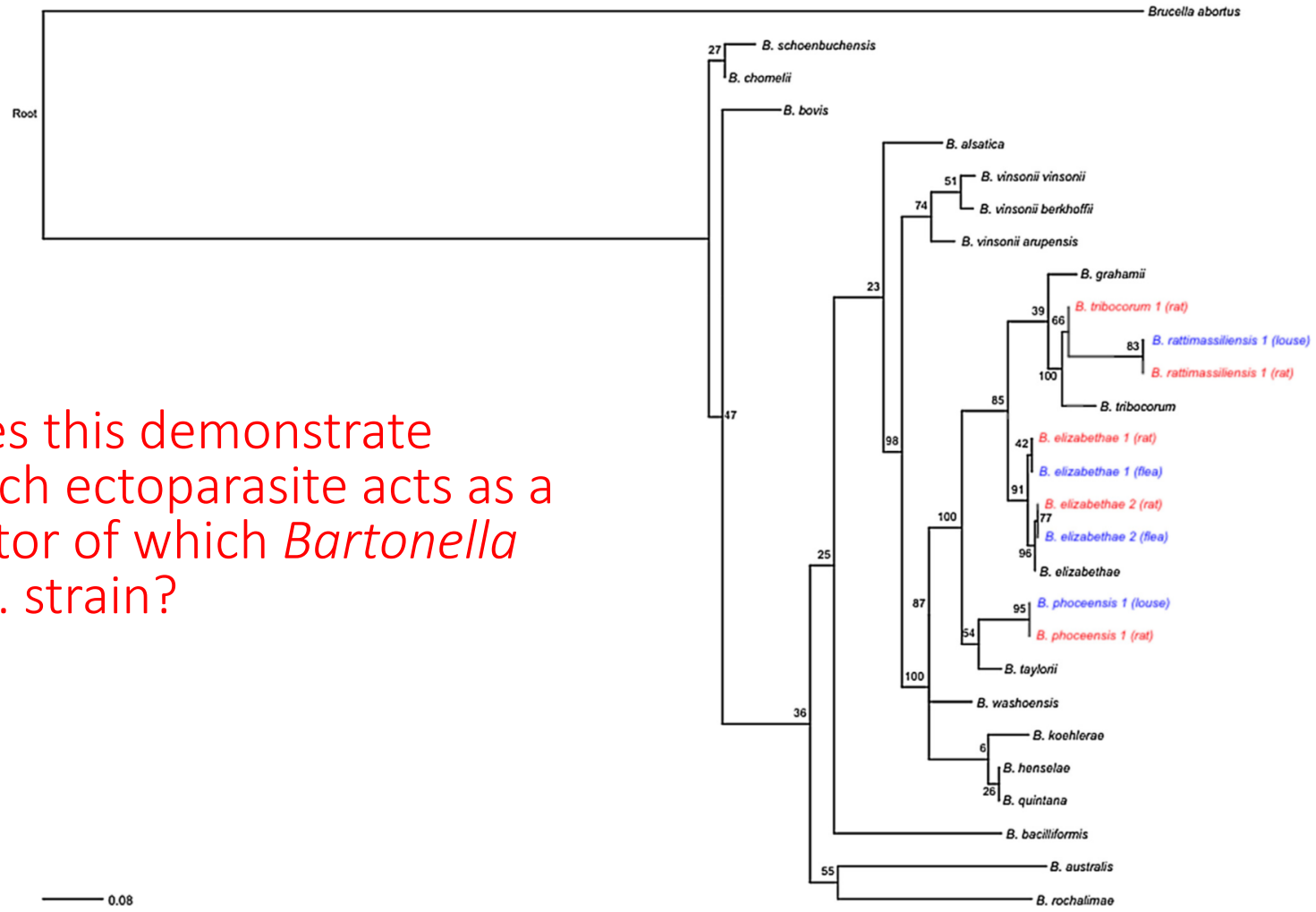
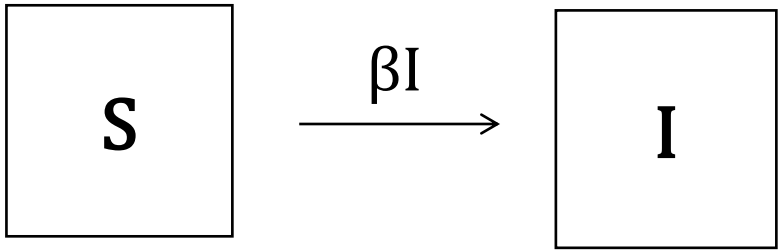


Does this demonstrate which ectoparasite acts as a vector of which *Bartonella* spp. strain?

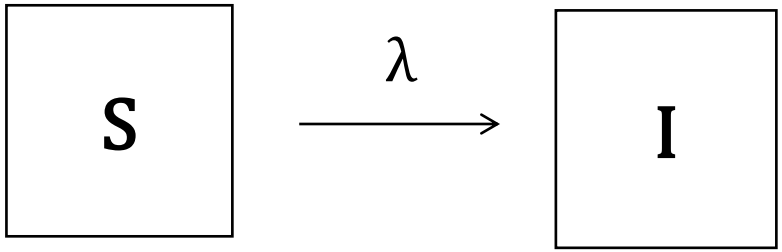
Then, we asked:
How does the rate of becoming infected vary with age?

Age-prevalence data allows for powerful inference into the dynamics of pathogen transmission.



for a persistent, non-immunizing infection

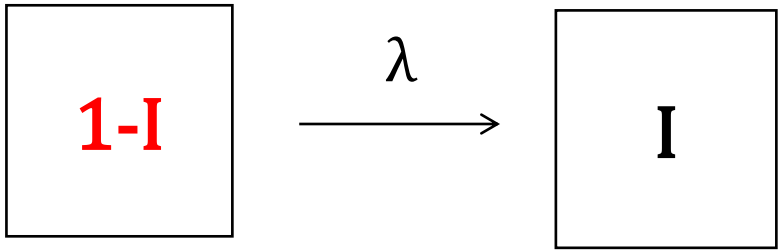
Age-prevalence data allows for powerful inference into the dynamics of pathogen transmission.



where λ, the force of infection, is the per capita rate at which susceptible hosts become infected

Age-prevalence data allows for powerful inference into the dynamics of pathogen transmission.

with a persistent infection, we can assume that, if not infected, you must be susceptible….



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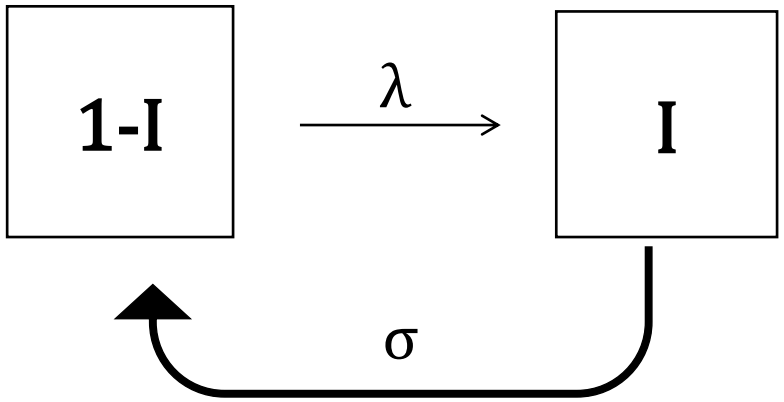
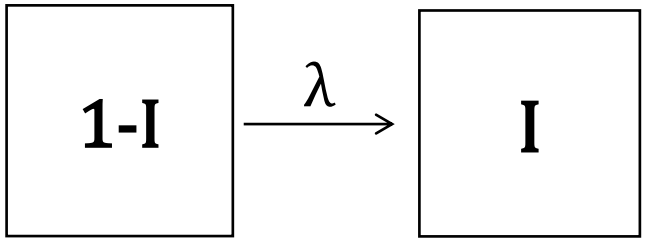
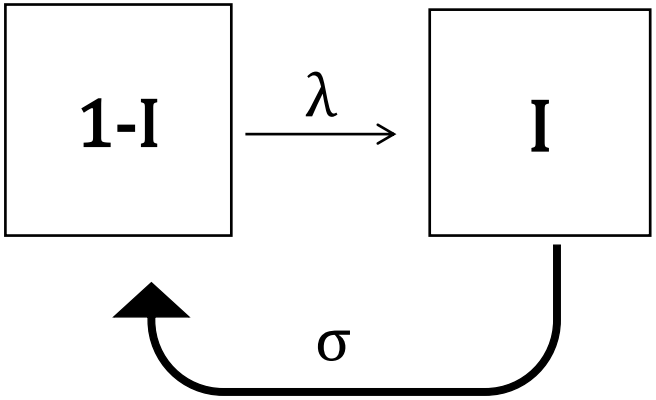


and σ is the rate of recovery from infection

Age-prevalence data allows for powerful inference into the dynamics of pathogen transmission.



$$\frac{dI(a)}{da} = \lambda(a)\left(1 - I(a)\right)$$



$$\frac{dI(a)}{da} = \lambda(a)\left(1 - I(a)\right) - \sigma I(a)$$

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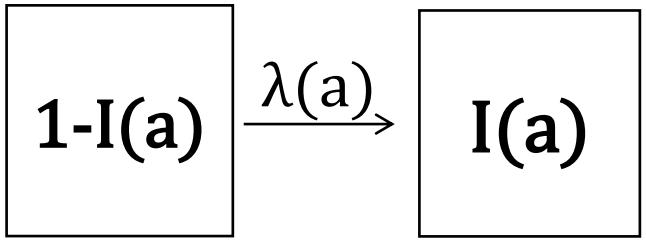

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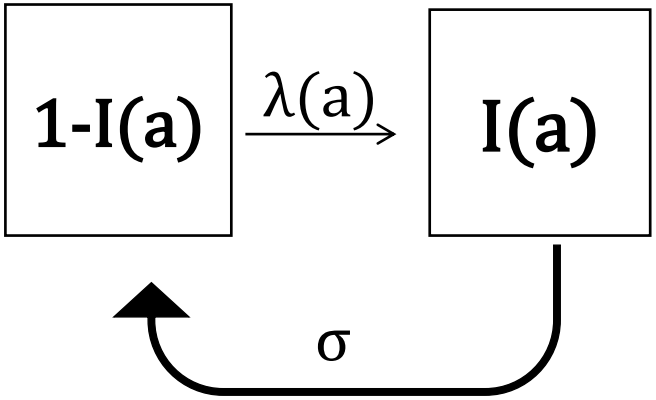

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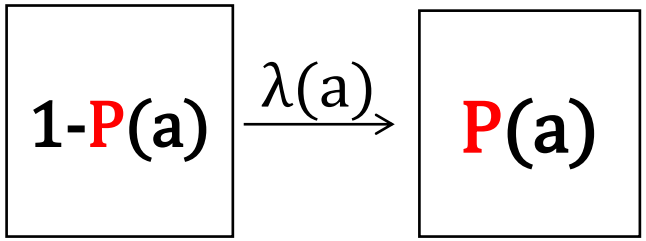
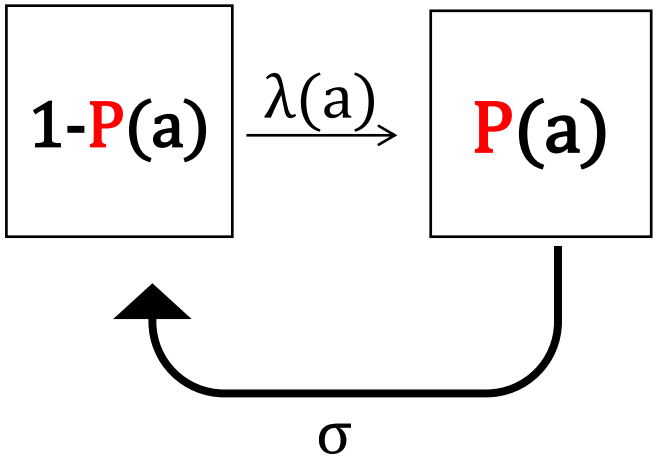


$$\frac{d\,P(a)}{da} = \lambda(a)\,(1 - P(a))$$



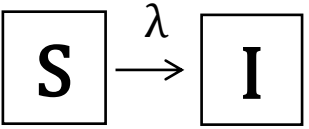
$$\frac{d\,P(a)}{da} = \lambda(a)\,(1 - P(a)) - \sigma(a)P(a))$$

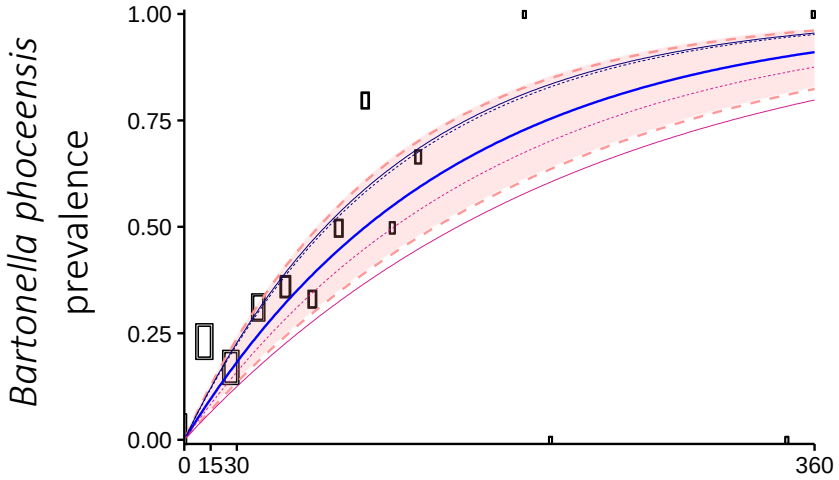
Compare using ***AIC = 2K - 2ln(L)***

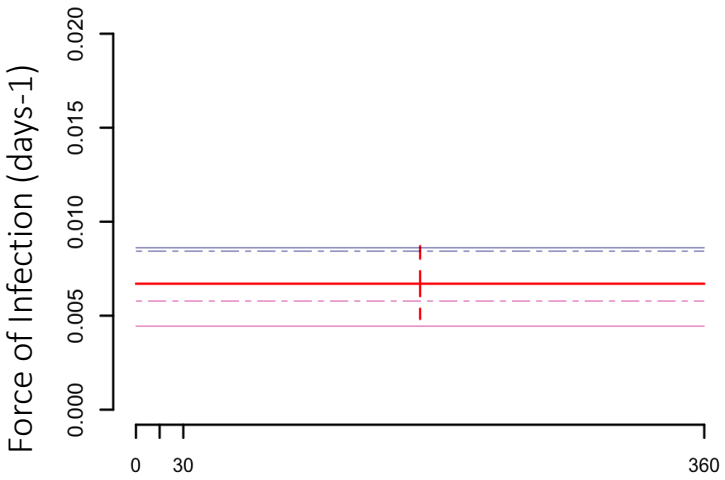
similar techniques can also be applied to age-seroprevalence data for immunizing infections

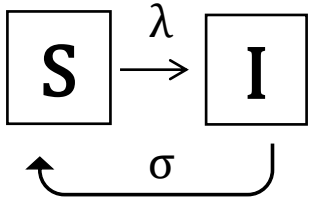
Let’s see which model works best for your data!

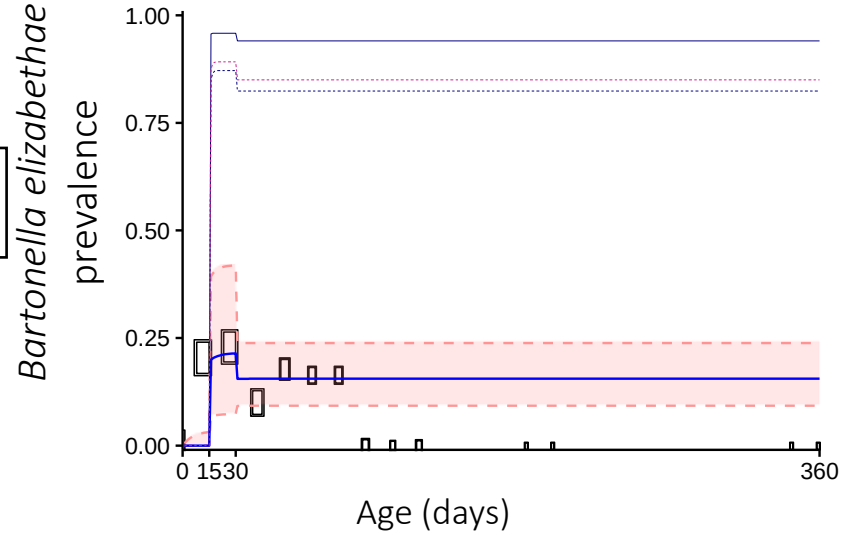
We found that an **SI model** offered the best fit to ***B. phoceensis*** data while the **SIS model** offered the best fit to the ***B. elizabethae*** data.

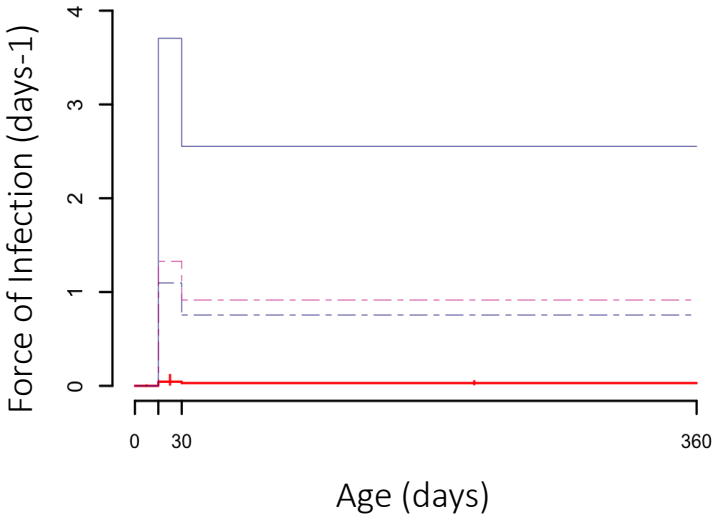




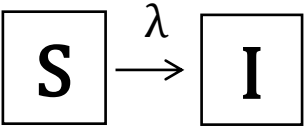


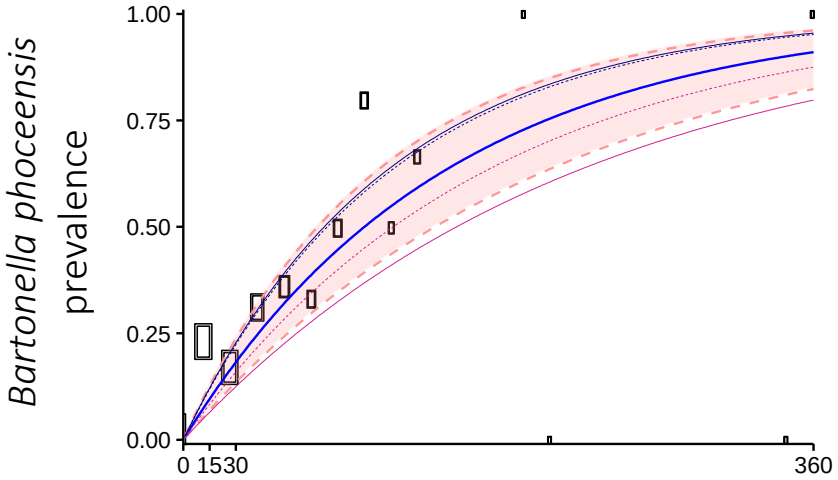


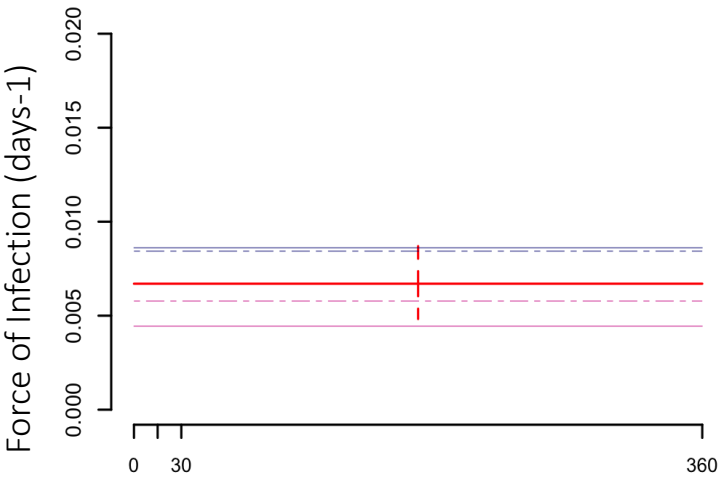


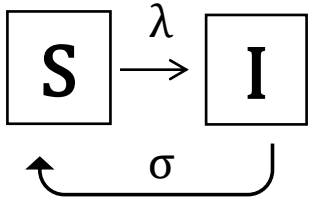


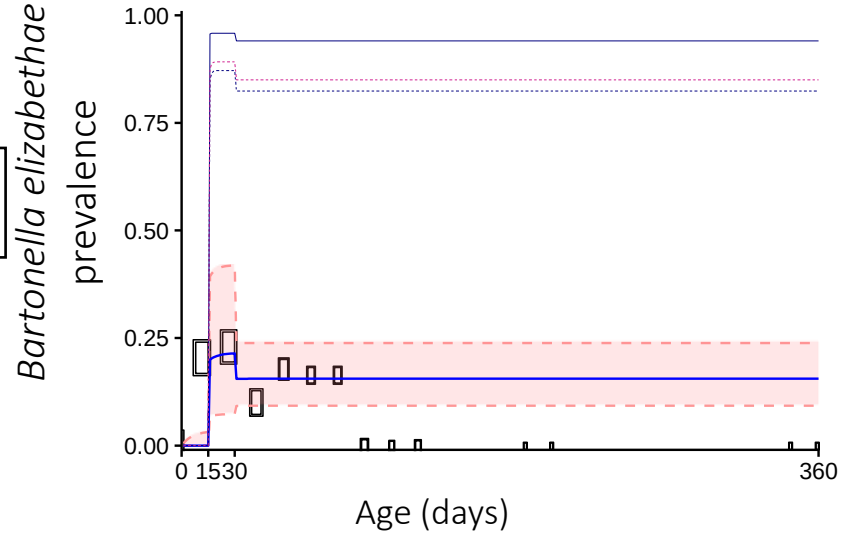
The age-structured FOI identifies age cohorts most influential in an epidemic. Juveniles showed the highest FOI.

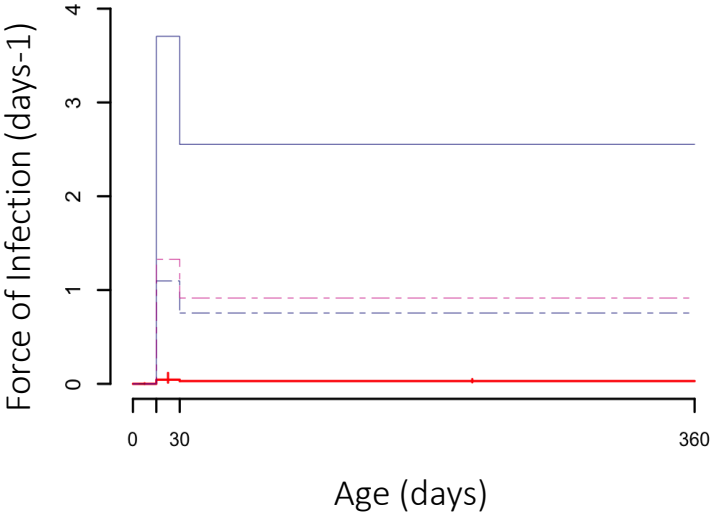


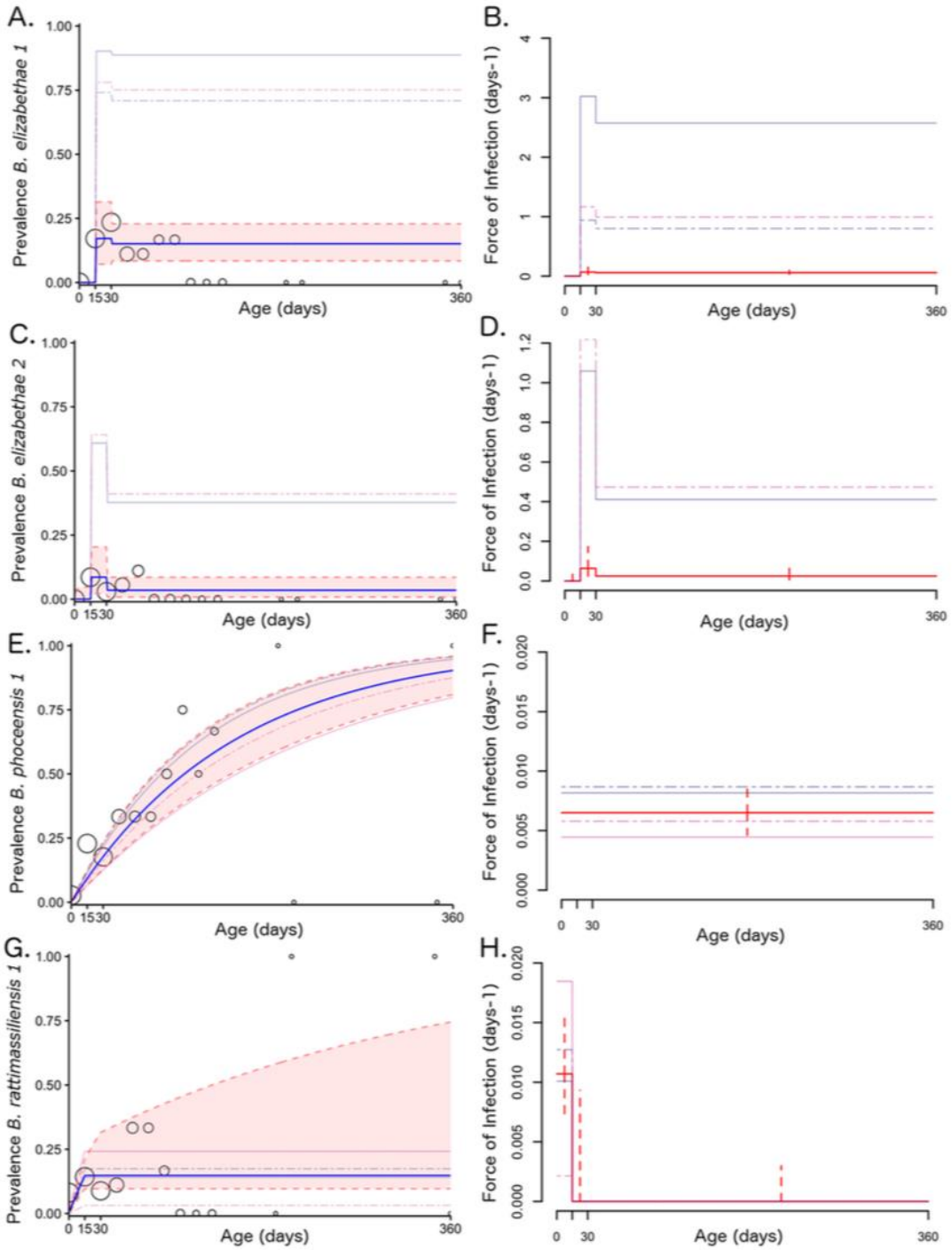











A.
Prevalence B. elizabethae 1
1.00
0.75
0.50
0.25
0.00
0 15 30
360
Age (days)
B.
Force of Infection (days-1)
4
3
2
1
0
0
30
360
Age (days)
C.
Prevalence B. elizabethae 2
1.00
0.75
0.50
0.25
0.00
0 15 30
360
Age (days)
D.
Force of Infection (days-1)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
30
360
Age (days)
E.
Prevalence B. phoceensis 1
1.00
0.75
0.50
0.25
0.00
0 15 30
360
Age (days)
F.
Force of Infection (days-1)
0.020
0.015
0.010
0.005
0.000
0
30
360
Age (days)
G.
Prevalence B. rattimassiliensis 1
1.00
0.75
0.50
0.25
0.00
0 15 30
360
Age (days)
H.
Force of Infection (days-1)
0.020
0.015
0.010
0.005
0.000
0
30
360
Age (days)