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Basic mathematics

E2M2 2020
Valbio Ranomafana
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@SarobidyRakoto

Fieldwork-based publications n conservation science decreased by 20% in comparison to a rise of 600% and 800% in modelling and data analysis studies, respectively. sciencedirect.com/science/articl..., the academic reward systems largely account for this trend!

Are fieldwork studies being relegated to second place in co...
The collection of biological information, including data gathered in the field, is fundamental to improve our ...
sciencedirect.com

1:21 PM · Aug 6, 2019 · Twitter Web App

67 Retweets **80** Likes

The Ferrari Lab @TheFerrariLab · Aug 9
Replying to @SarobidyRakoto
If modelling and data analysis studies aren't leading to better, more efficient, more effective data collection (in the field, in the lab, wherever) then we're really just standing around staring at our toes -- signed, someone who does modeling and data analysis

1 1 6

Julia P G Jones
@juliapgjones

Why do so many students say 'I can't do maths' when faced with a simple calculation (this was 10,000*40). It's like not doing maths is part of their identity & barely look before saying they can't. What can we do to help them see a) they can, b) there is help worth engaging with?

5:01 PM · Nov 19, 2019 · Twitter for Android

4 Retweets **21** Likes

Isabel Rosa @isamdr86 · Nov 19, 2019
Replying to @juliapgjones
I wonder about the same... how do they go about understanding the world around them without basic maths?

1

Julia P G Jones @juliapgjones · Nov 19, 2019
My concern isn't really about skill level (that can be addressed) but the belief they can't do maths- this is so limiting. People aren't usual proud to say 'I can't read' but will happily report not doing maths as if it's an immutable character/ something to take pride in.

The goal of this session is to give you tools to
(1) be able to do basic mathematics
(2) understand equations in papers

Outline

- Greek letters
- Order of operations and parentheses
- Common mathematical notations
- Matrix operation
- Function
- Difference and differential equation

Greek letters

Jody Reimer

$$\frac{dx}{dt} = rx\left(\frac{x}{\alpha K} - 1\right)\left(1 - \frac{x}{K}\right)$$

What is alpha? The one like the fish

Greek letters

Α α	Β β	Γ γ	Δ δ	Ε ε	Ζ ζ	Η η	Θ θ
ἄλφα	βῆτα	γάμμα	δέλτα	ἔψιλόν	ζῆτα	ἦτα	θῆτα
alpha	beta	gamma	delta	epsilon	zeta	eta	theta
a	b	g	d	e	z	ē	th
[a/aː]	[b]	[g]	[d]	[e]	[zd/dz]	[ɛː]	[tʰ]
Ι ι	Κ κ	Λ λ	Μ μ	Ν ν	Ξ ξ	Ο ο	Π π
ἰῶτα	κάππα	λάμβδα	μῦ	νῦ	ξεῖ	ὄμικρόν	πεῖ
iota	kappa	lambda	mu	nu	xi	omikron	pi
i	k	l	m	n	ks/x	o	p
[i/iː]	[k]	[l]	[m]	[n]	[ks]	[o]	[p]
Ρ ρ	Σ σ/ς	Τ τ	Υ υ	Φ φ	Χ χ	Ψ ψ	Ω ω
ῥῶ	σῖγμα	ταῦ	ὕψιλόν	φεῖ	χεῖ	ψεῖ	ὦμέγα
rho	sigma	tau	upsilon	phi	chi	psi	omega
r/rh	s	t	u/y	ph	kh/ch	ps	ō
[r]	[s/z]	[t]	[y/yː]	[pʰ]	[kʰ]	[ps]	[ɔː]

Order of operation and parentheses

$$8 \div 2(2 + 2) = ?$$

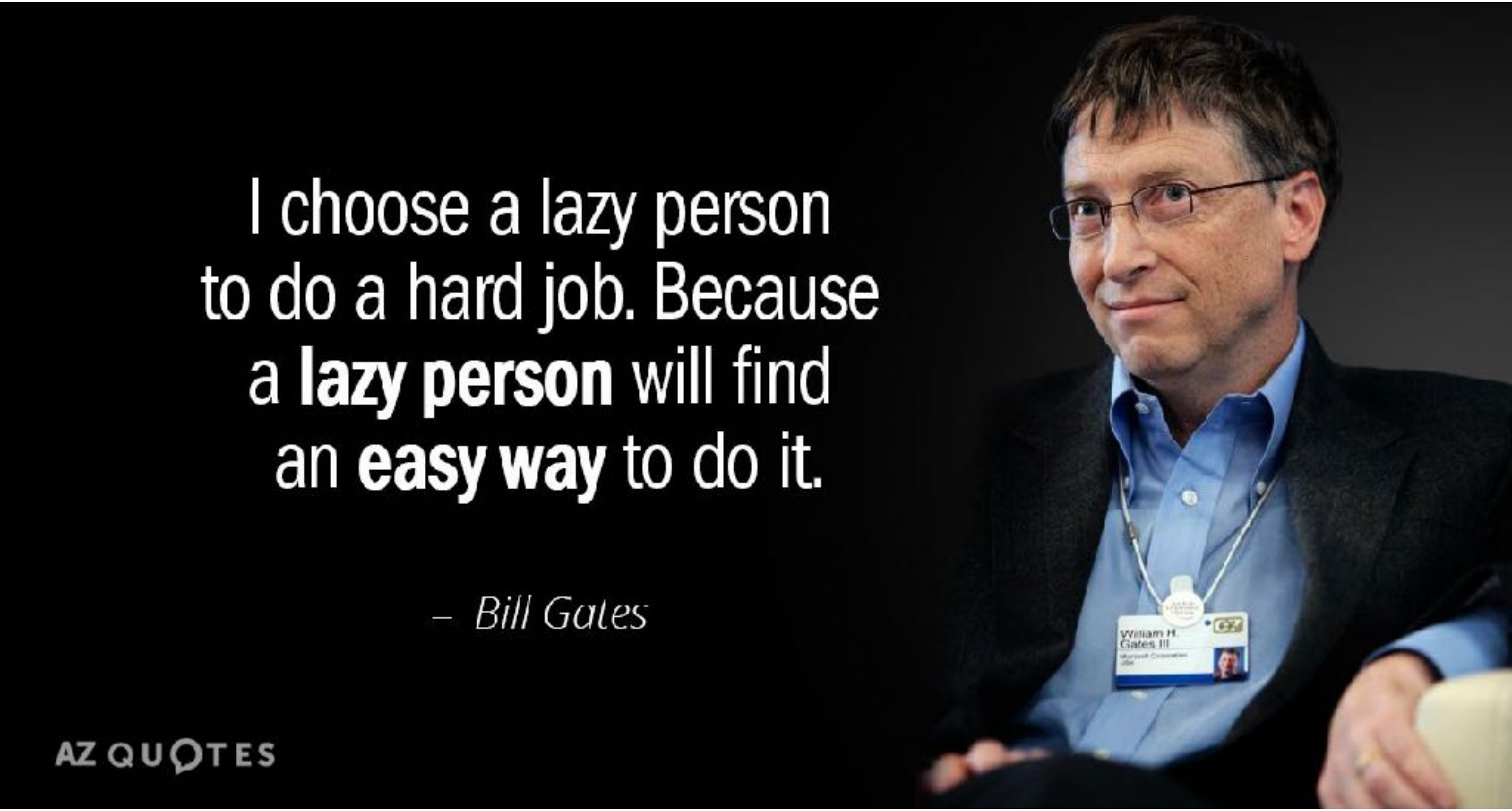
Orders of operations

- $\{+, -\} < \{\cdot, \times, *, \div, :\} < \{\text{^}, \text{power}\} < (), \{\}, []$
- In case of a doubt, always use parentheses

Examples

- 1+ 1/2 + 3
- (1+1)/2 + 3
- 1+1/(2+3)
- (1+1)/(2+3)
- 1/2/3

- x+ y/2 + z
- (x+y)/2 + z
- x+y/(2+z)
- (x+y)/(2+z)
- x/2/z


I choose a lazy person
to do a hard job. Because
a **lazy person** will find
an **easy way** to do it.
– Bill Gates
AZ QUOTES

Sums

- $1 + 2 + 3 + 4 = ?$
- $1^2 + 2^2 + 3^2 + 4^2 = ?$
- $1^1 + 2^2 + 3^3 + 4^4 = ?$

Sums

- $$\sum_{i=1}^{4} i = 1 + 2 + 3 + 4$$
- $$\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2$$
- $$\sum_{i=1}^{4} i^i = 1^1 + 2^2 + 3^3 + 4^4$$

Products

- $1 \times 2 \times 3 \times 4 = ?$
- $1^2 \times 2^2 \times 3^2 \times 4^2 = ?$
- $1^1 \times 2^2 \times 3^3 \times 4^4 = ?$

Products

- $$\prod_{i=1}^{4} i = 1 \times 2 \times 3 \times 4 = 4!$$
- $$\prod_{i=1}^{4} i^2 = 1^2 \times 2^2 \times 3^2 \times 4^2$$
- $$\prod_{i=1}^{4} i^i = 1^1 \times 2^2 \times 3^3 \times 4^4$$

Some ideas

- Recognize dummy variables and don’t be afraid of them
- Notations should be intuitive and consistent

Matrix

$$M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$

- $M + \alpha = \alpha + M = ?$
- $M \cdot 2 = 2 \cdot M = ?$

- **Matrix multiplication**
- $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = ?$
- $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$
- What is the R syntax?

Matrix

- Inverse of a matrix: $M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix}$ is $M^{-1} = \begin{bmatrix} 1 & -1/3 & 1/3 \\ 0 & 1/6 & -1/6 \\ 0 & 1/3 & 2/3 \end{bmatrix}$
- **THERE IS NO MATRIX DIVISION**
- Identity matrix $\mathrm{Id} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Diagonal matrix $M = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Transpose of a matrix: $M^t = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

Eigenvectors and eigenvalues

$$M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix} \quad v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

- $M \cdot v_1 = ?$
- $M \cdot v_2 = ?$
- $M \cdot v_3 = ?$

Eigenvectors and eigenvalues

$$M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix} \quad v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

- $M \cdot v_1 = 3v_1$
- $M \cdot v_2 = 2v_2$
- $M \cdot v_3 = 1v_3$

Eigenvectors and eigenvalues

$$M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & 1 \\ 0 & -2 & 1 \end{bmatrix} \quad v_1 = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

- $M \cdot v_1 = 3v_1$
- $M \cdot v_2 = 2v_2$
- $M \cdot v_3 = 1v_3$

If $V = [v_1\ v_2\ v_3] = \begin{bmatrix} -1 & -2 & 1 \\ -1 & -1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

and $\Lambda = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $M = V^{-1} \Lambda V$

2- Déterminer les droites Δ telles que $f(\Delta)$ soient parallèles à Δ.

PROBLEME 2

On considère la fonction f définie sur l'intervalle $[0;+\infty[$ par $f(x)=\ln\left(e^{x}+e^{-x}\right)$.

On désigne par $(\mathscr{C})$ sa courbe représentative dans un repère orthonormé $(O;\vec{i},\vec{j})$ d'unité 3 cm.

Partie A

1- a) Déterminer la limite de f en $+\infty$.

b) Montrer que, pour tout $x\in[0;+\infty[$ on a :

$f(x)=x+\ln\left(1+e^{-2x}\right)$.

c) En déduire que la courbe $(\mathscr{C})$ admet comme asymptote la droite (Δ) d'équation $y=x$.

d) Etudier la position relative de $(\mathscr{C})$ et (Δ).

2- Etudier le sens de variation de f et dresser son tableau de variation.

3- Tracer la droite (Δ) et la courbe $(\mathscr{C})$.

4- On considère l'équation différentielle (E) : $y''-y=0$.

a) Résoudre l'équation (E) sur $[0;+\infty[$.

b) Déterminer la fonction h, solution de l'équation (E) qui vérifie $h(0)=2$ et $h'(0)=0$.

c) Vérifier que pour tout $x\in[0;+\infty[$, $f(x)=\ln h(x)$.

Partie B

Pour tout $x\in[0;+\infty[$ on pose $F(x)=\int_0^x \ln\left(1+e^{-2t}\right)dt$.

On ne cherchera pas à calculer $F(x)$.

1- Soit x un réel strictement positif. En utilisant la question 1- de la partie A, donner une interprétation géométrique de $F(x)$.

2- Etudier le sens de variation de F sur l'intervalle $[0;+\infty[$.

3- Soit a un réel strictement positif.

a) Montrer que, pour tout $t\in[1;1+a]$ on a $\frac{1}{1+a}\leq\frac{1}{t}\leq 1$.

Problème	**12 points**

Soit f la fonction définie sur l'intervalle $]-4;2[$ par :
$f(x)=\ln(x+4)-\ln(2-x)$.

On note par $(\mathscr{C})$ la courbe représentative de f dans un repère orthonormé $(O;\vec{i},\vec{j})$, d'unité 2 cm.

1. Calculer les limites de f en -4 et en 2. Interpréter graphiquement ces résultats. *(2 pts)*
2. a. Montrer que, pour tout $x\in]-4;2[$, la fonction dérivée de f est :
   $f'(x)=\frac{6}{(2-x)(x+4)}$. *(1 pt)*
   b. Dresser le tableau de variation de f. *(1 pt)*
3. a. Déterminer le point d'intersection de $(\mathscr{C})$ avec l'axe des abscisses. *(1 pt)*
   b. Ecrire l'équation de la tangente (T) à $(\mathscr{C})$ au point $I(-1;0)$. *(1 pt)*
   c. Montrer que le point $I(-1;0)$ est un centre de symétrie pour $(\mathscr{C})$. *(1 pt)*
4. Tracer (T) et $(\mathscr{C})$ dans un même repère. *(2 pts)*

Soit F la fonction définie sur l'intervalle $]-4;2[$ par :

$F(x)=(x+4)\ln(x+4)-(x-2)\ln(2-x)$.

a. Calculer la fonction dérivée F' de F. *(1 pt)*
b. En déduire la valeur exacte en cm^2 de l'aire du domaine plan limité par $(\mathscr{C})$, l'axe des abscisses et les droites d'équations $x=-1$ et $x=0$. *(1 pt)*

6. Soit g la fonction définie sur $]-4;2[$ par :
   $g(x)=\ln\left(\frac{2-x}{x+4}\right)$.
   a. Montrer que, pour tout $x\in]-4;2[$: $g(x)=-f(x)$. *(0,5 pt)*
   b. Tracer dans le même repère que $(\mathscr{C})$ la courbe représentative (Γ) de g. *(0,5 pt)*

Functions

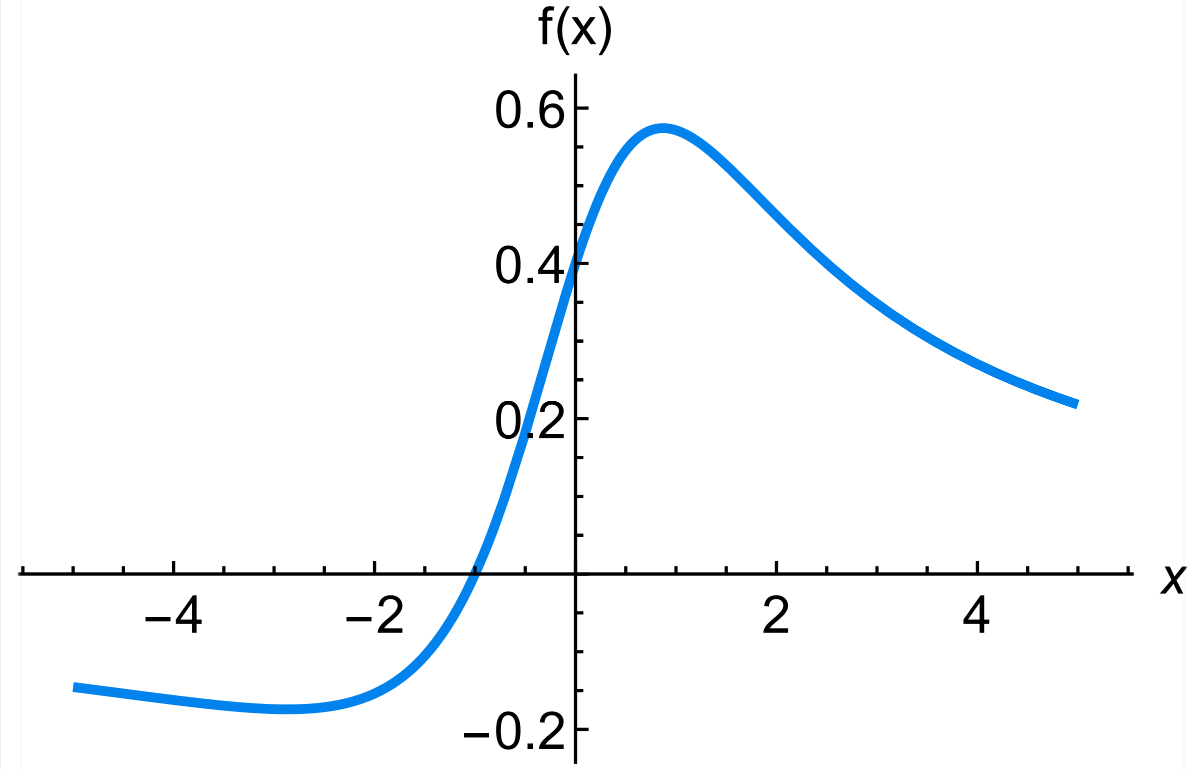
With a minimal effort, you can use R to plot the curve of any function, what matters now is:
Can you read/interpret the curve?

$$f(x) = \frac{x+1}{x^2+2.5} \text{ for } x \in [-5,5]$$

Function properties

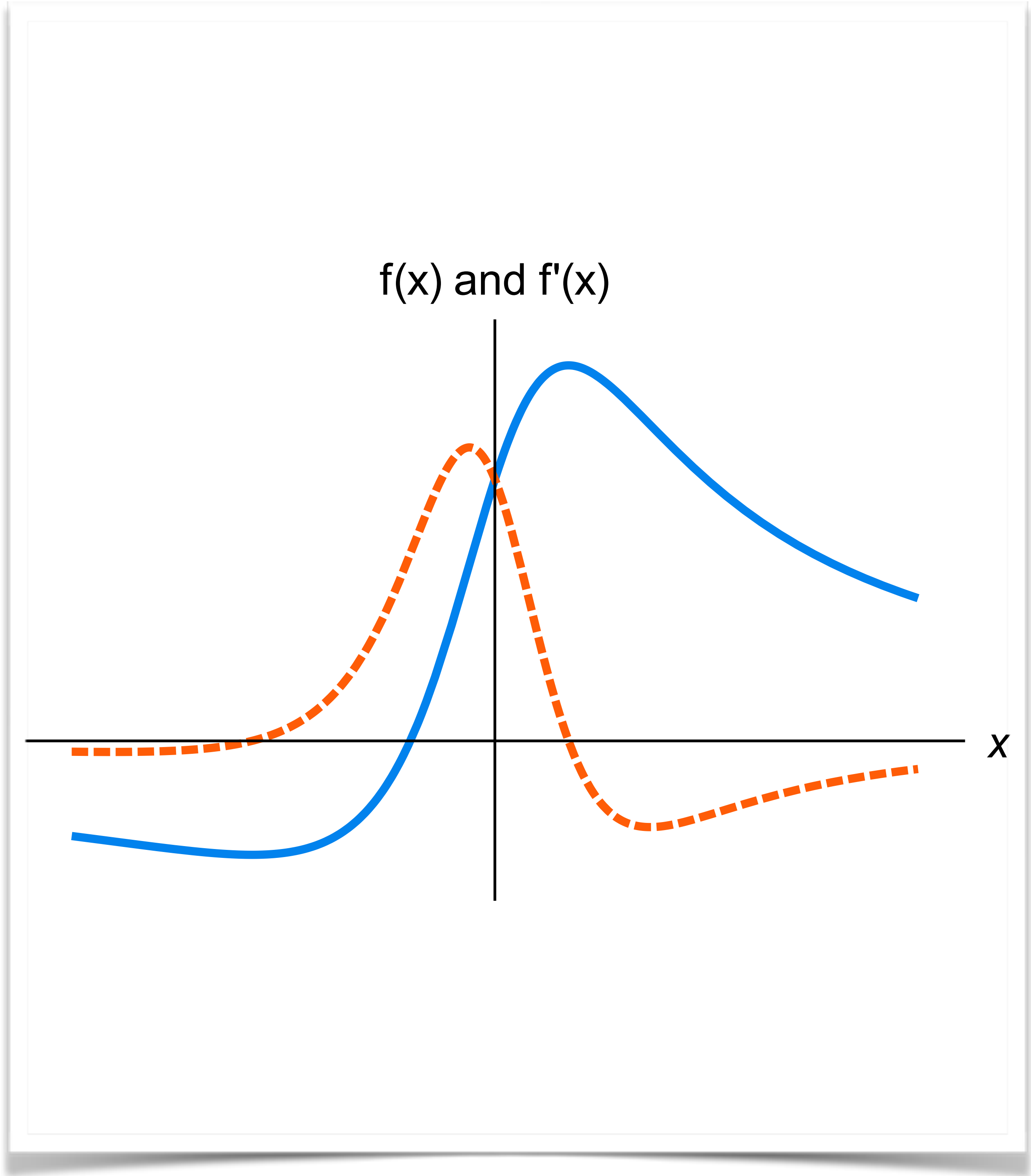
- Intercept / root(s)
- Positive / negative value
- Maximum / minimum value
- Increasing / Decreasing / Constant
- Concave / Convexe
- Asymptotic

$$f(x) = \frac{x+1}{x^2+2.5}$$



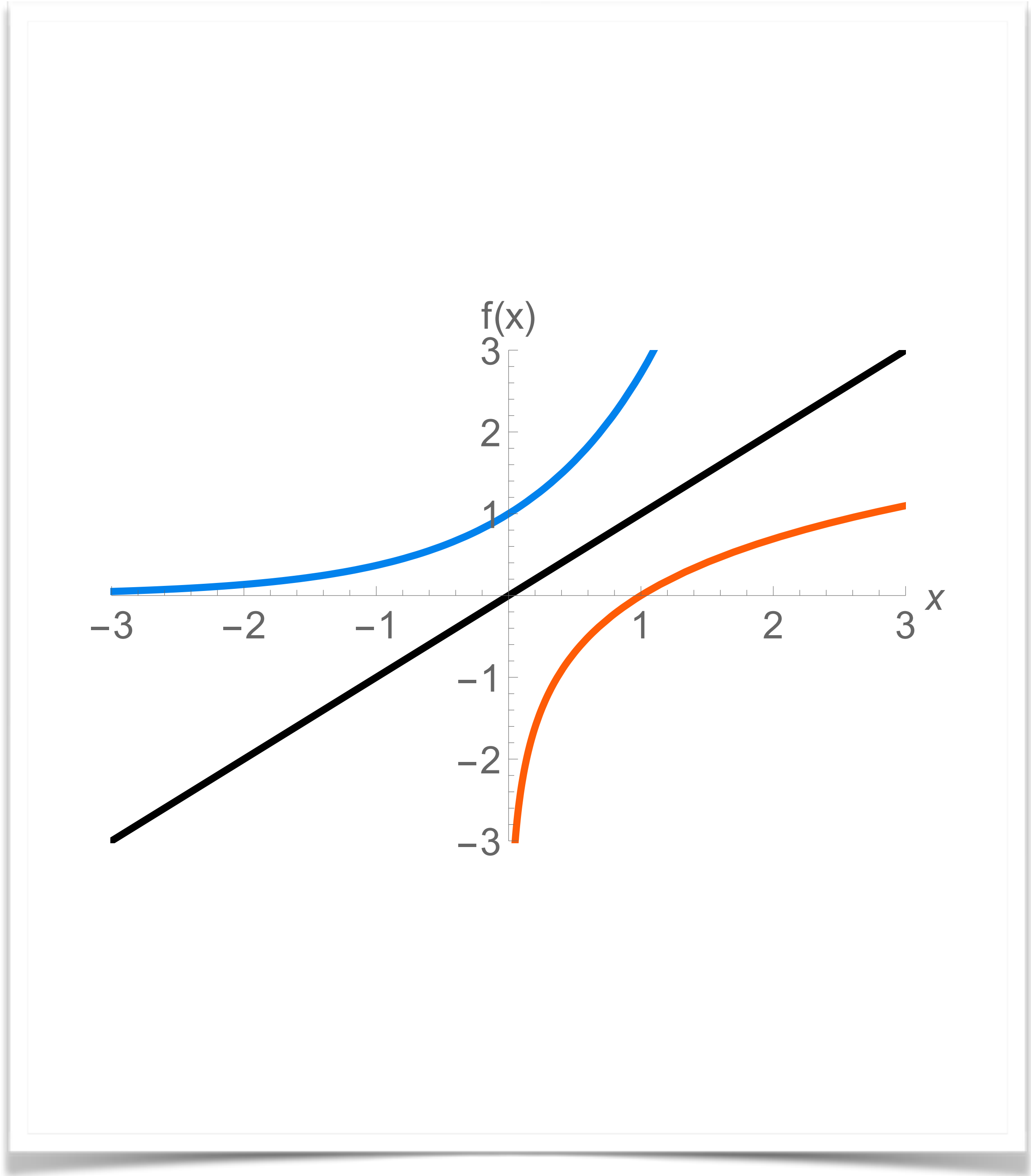
A function and its derivative

- What happen when the derivative is:
  - negative?
  - positive?
  - zero?
  - reaching a maximum (finite) value?
- Homework
  - The derivative is increasing
  - The derivative is decreasing



Special functions

- $f(x) = \ln(x)$
  - $\ln(ab) = \ln(a) + \ln(b)$
  - $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
  - $\ln(a^n) = n\ln(a)$
- $f(x) = e^x$
  - $e^a e^b = e^{a+b}$
  - $\frac{e^a}{e^b} = e^{a-b}$
  - $\left(e^x\right)^n = e^{nx}$
- $\ln(e^x) = x\ln(e) = x = e^{\ln(x)}$



“A picture is worth a thousand words. ”

–Fred R. Barnard

"A picture is worth a thousand words,
but an equation is worth a thousand pictures"

–Fred R. Barnard and someone else

Difference and differential equations

Difference equation (suite in french)

- Explicit: $u_n = f(n)$
  - E.g., $u_n = 2^n$, $u_n = e^n - \log(n) + \dfrac{\sin(n^2)}{\sqrt{n+2}}$
- Implicit: $u_n = f(u_{n-1}, u_{n-2}, \ldots)$
  - E.g., $u_n = u_{n-1} + bu_{n-1} - du_{n-1}$
- Just write the code and you will see the behavior!!!

Differential equation

- $$\frac{dx}{dt} = rx\left(\frac{x}{\alpha K} - 1\right)\left(1 - \frac{x}{K}\right)$$
- And more generally $\frac{dx}{dt} = f(x)$
- The equation simply describes how x change when t change