Department of
Integrative Biology
UNIVERSITY OF WISCONSIN–MADISON

Metapopulation Research Group

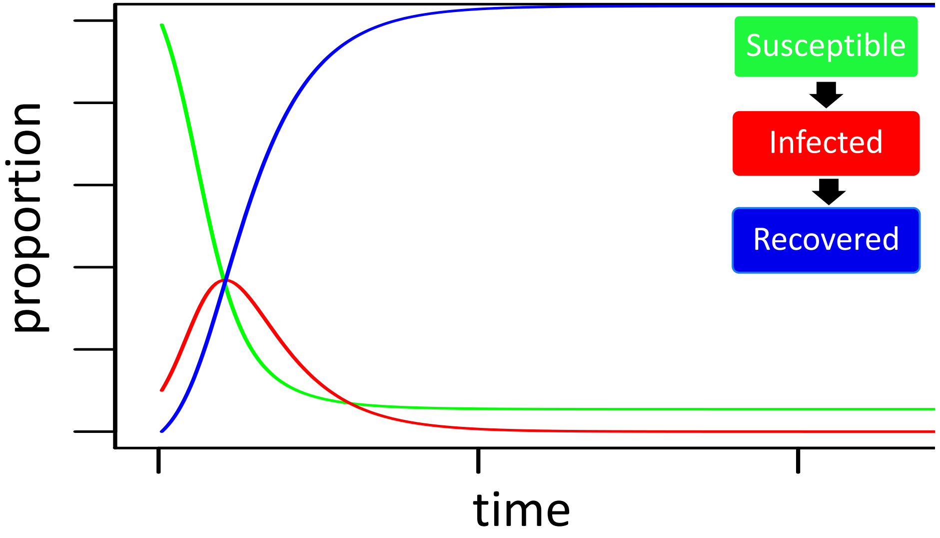
Spatial mechanistic models

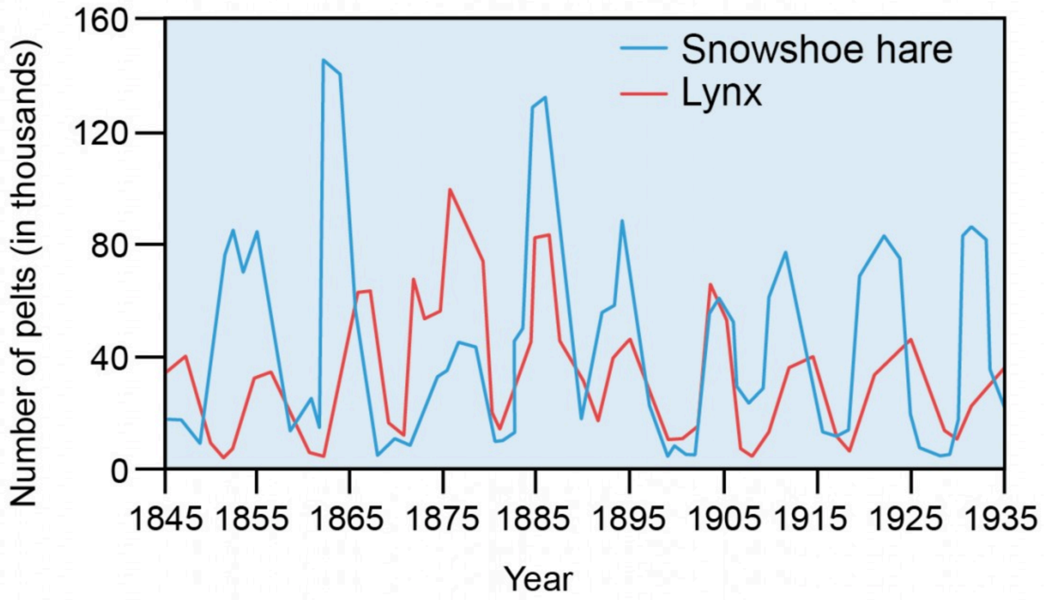
Tanjona Ramiadantsoa

with materials from Atte Molainen

Previously

- Variable of interest is population size through time



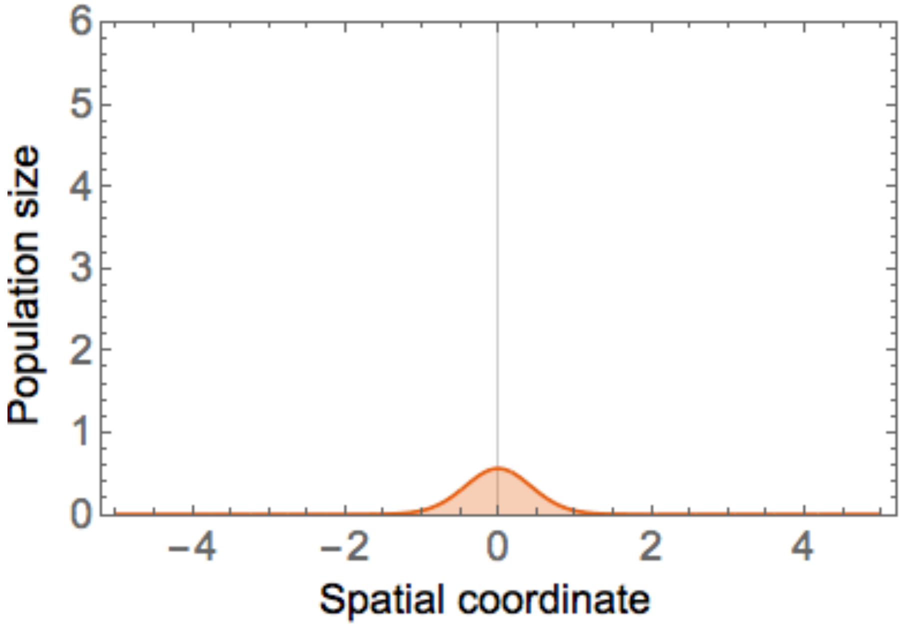


Spatial model

- Now, we are interested in population size **both through time and through space**

Spatial model

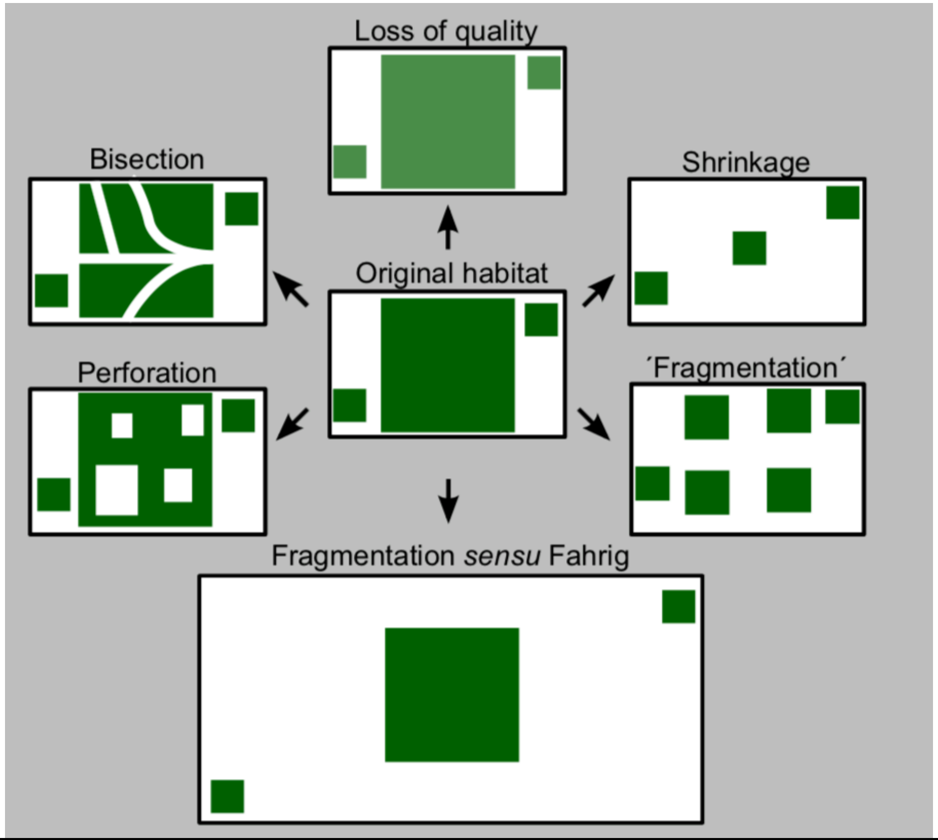
- Now, we are interested in population size **both through time and through space**



Spatial model

Two necessary additions

Landscape structure



Dispersal

Outline

- Metapopulation paradigm
- Levins’ (spatially implicit) metapopulation model
- Spatially realistic metapopulation model
- Software demonstration: SPOMSIM

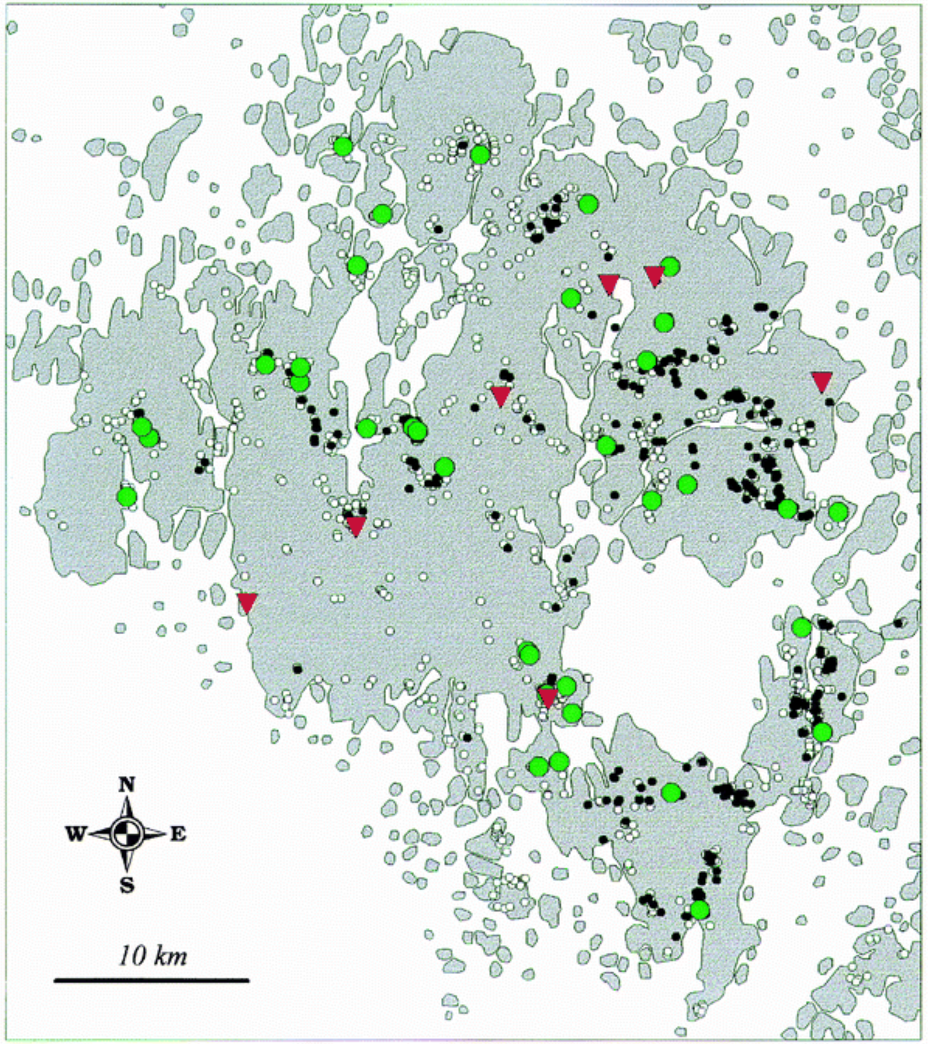
Outline

- Metapopulation paradigm
- Levins' (spatially implicit) metapopulation model
- Spatially realistic metapopulation model
- Software demonstration: SPOMSIM

Model system for metapopulation

Glanville fritillary butterfly
(*Melitaea cinxia*)

Photo: Hannu Aarnio

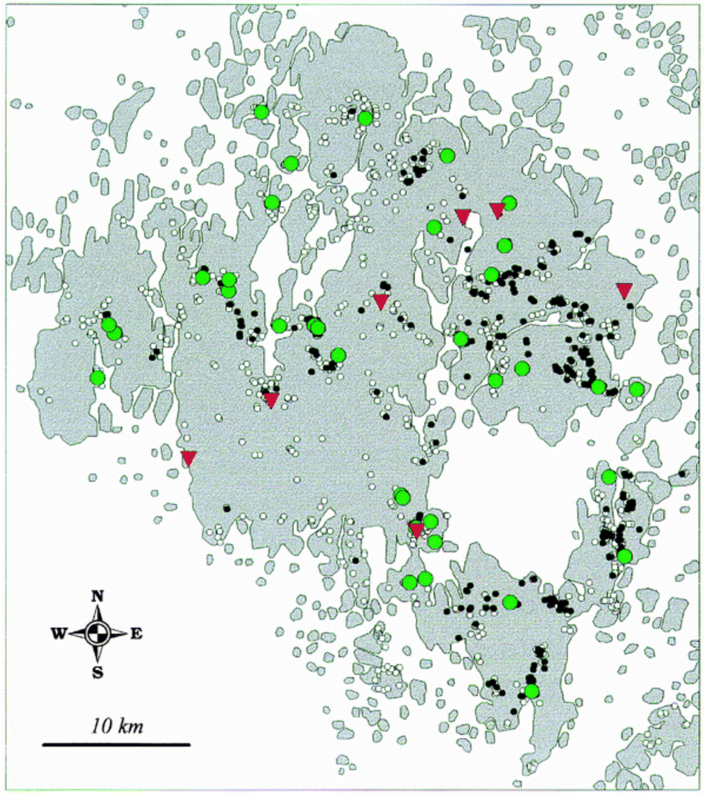


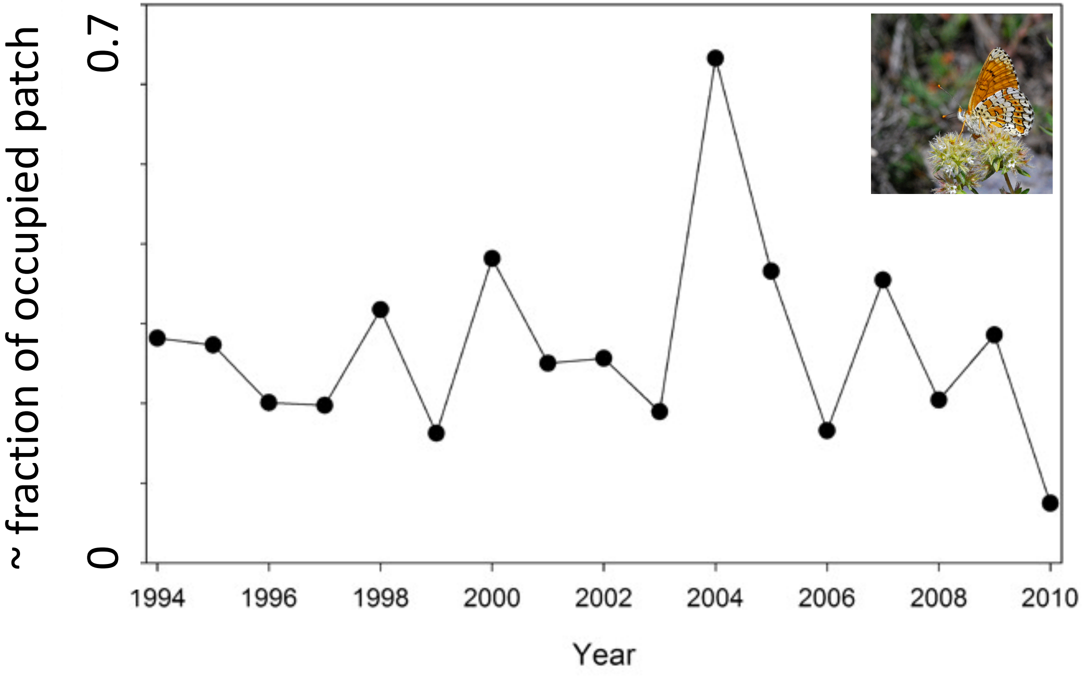
Saccheri et al. 1998

Ilkka Hanski

Metapopulation Research Group

Metapopulation dynamics





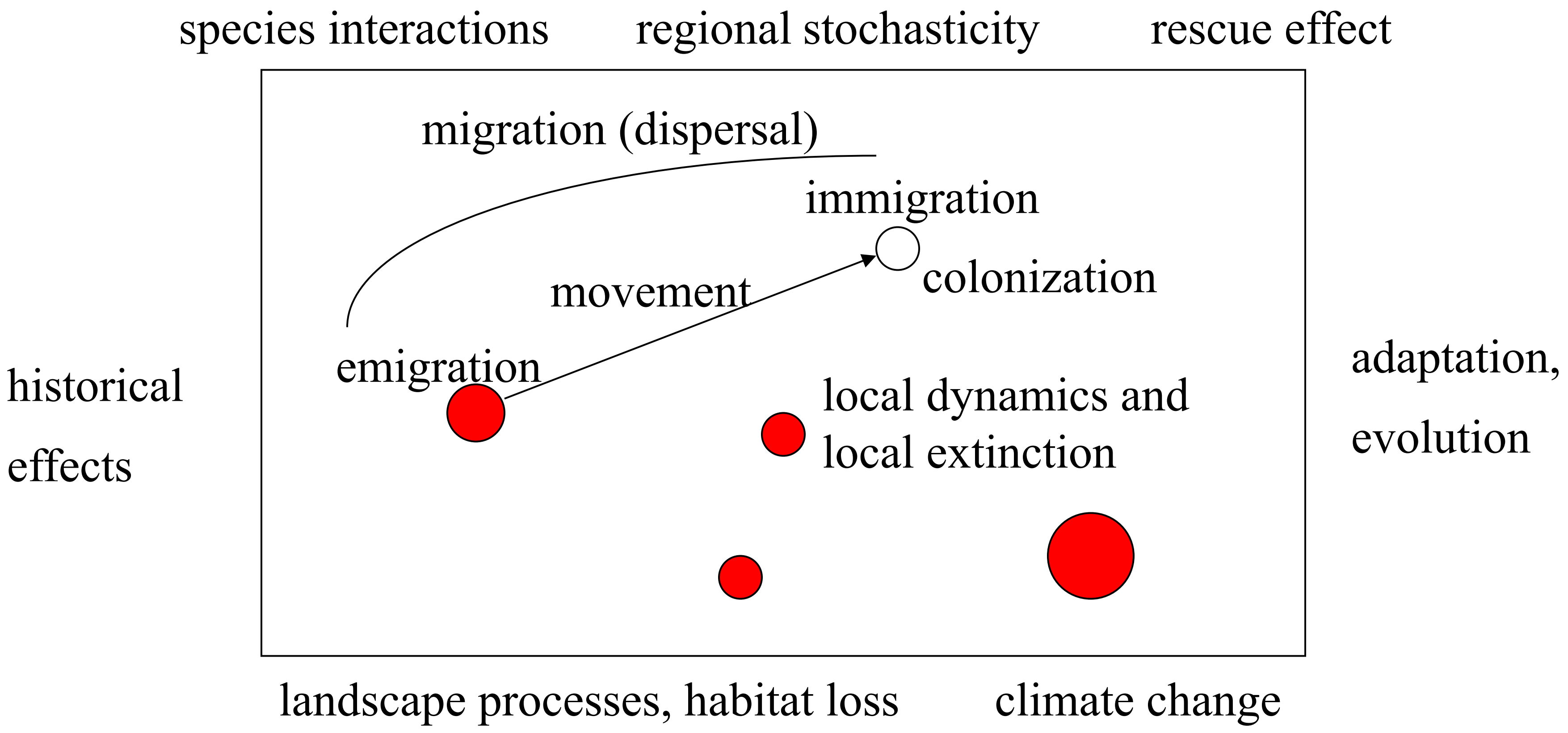
Hanski 2011

Metapopulation paradigm

Historically, ecologists mainly focus on local processes

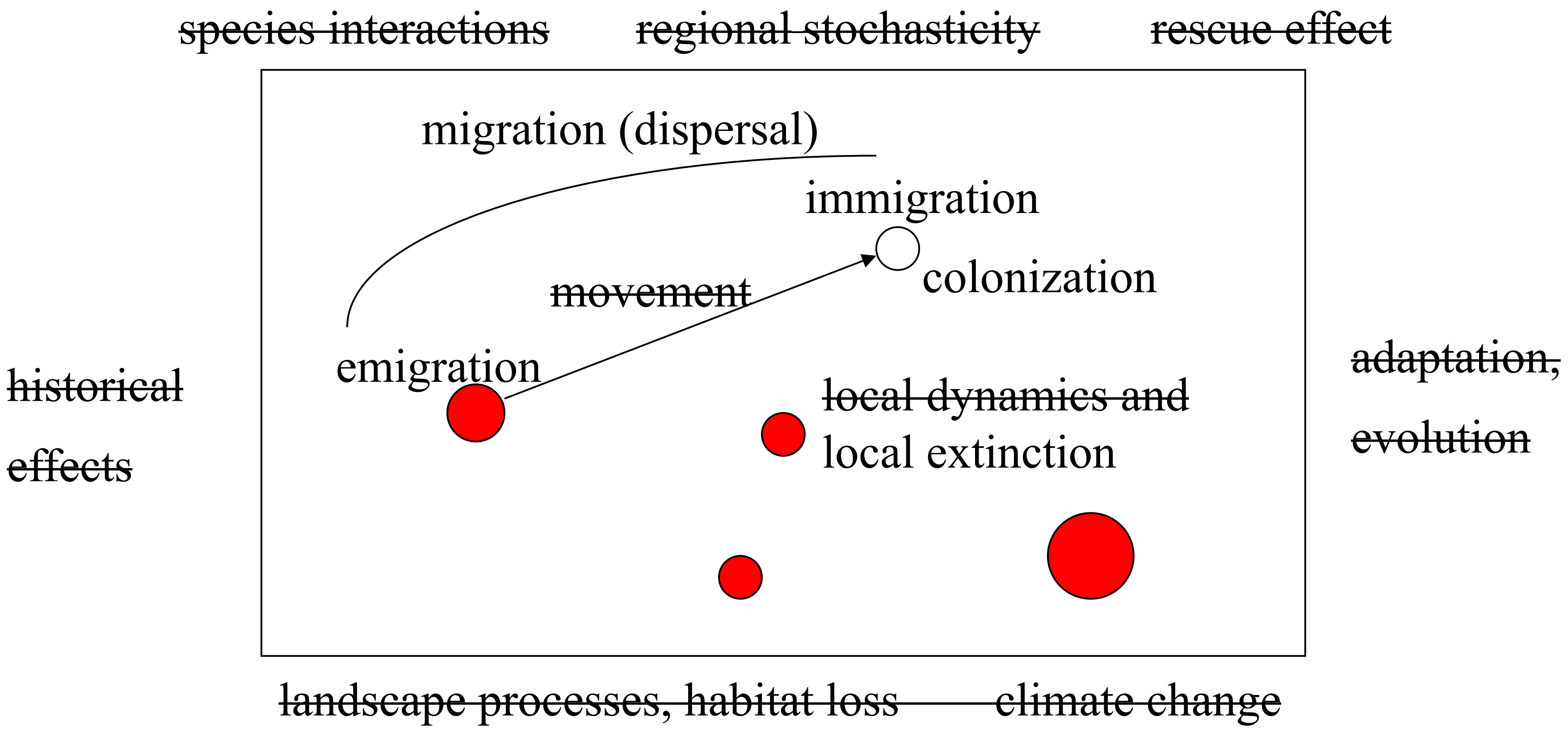
Metapopulation paradigm

Historically, ecologists mainly focus on local processes



Metapopulation paradigm

Historically, ecologists mainly focus on local processes

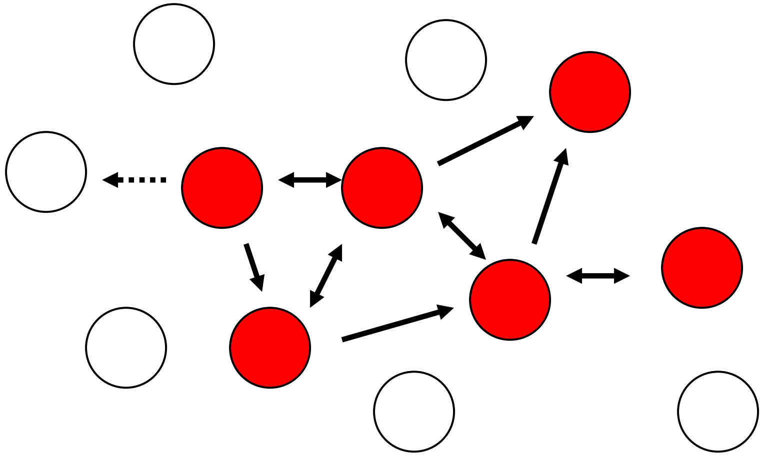


Outline

- Metapopulation paradigm
- Levins' (spatially implicit) metapopulation model
- Spatially realistic metapopulation model
- Software demonstration: SPOMSIM

The concept of metapopulation (Levins 1969)

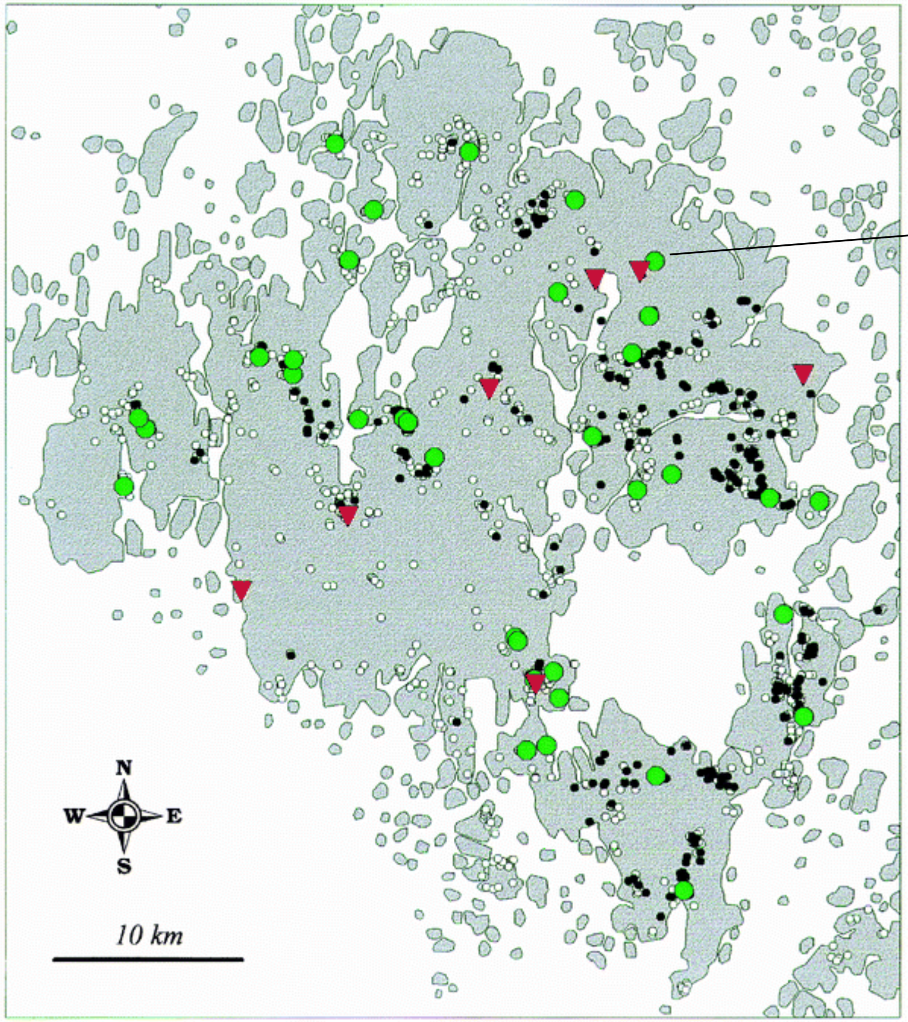
Richard Levins

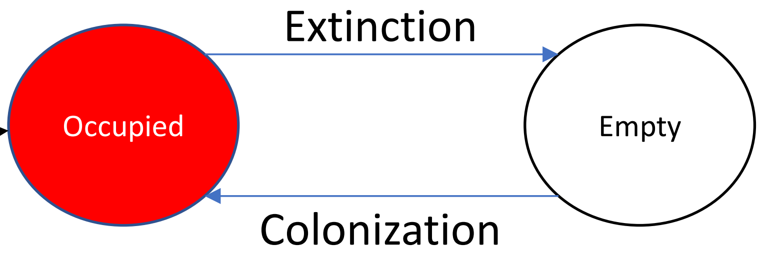


⇒ We only ask whether the patch is empty or occupied (occupancy model)

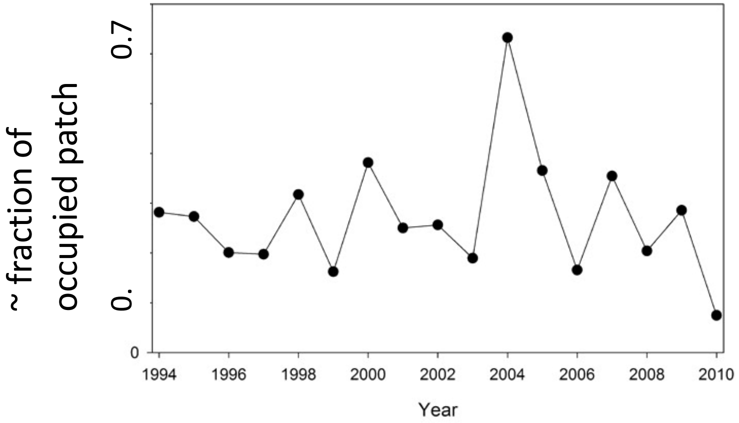
“a set of local populations connected by migration”

Conceptual model





We are interested in the fraction of occupied patch *p*



Levins' (spatially implicit) deterministic model

Spatial implicit model
1- All patches have the same probability of going extinct
2- All patches are equidistant to each other
3- Infinitely many patches (removes stochasticity)

Let ***p*** the fraction of occupied patch in the landscape
- Extinction happens at a rate *e*
- Colonization happens at a rate *cp*

$$\frac{dp}{dt} = \underbrace{cp\,\overbrace{(1-p)}^{\text{Empty patches}}}_{\text{All colonizations}} - \underbrace{e\,p}_{\text{All extinctions}}$$

Levins' (spatially implicit) deterministic model

Spatial implicit model
1- All patches have the same probability of going extinct
2- All patches are equidistant to each other
3- Infinitely many patches (removes stochasticity)

Let $\boldsymbol{p}$ the fraction of occupied patch in the landscape
- Extinction happens at a rate *e*
- Colonization happens at a rate *cp*

$$\frac{dp}{dt} = \underbrace{cp\overbrace{(1-p)}^{\text{Empty patches}}}_{\text{All colonizations}} - \underbrace{e\,p}_{\text{All extinctions}}$$

- Equilibria

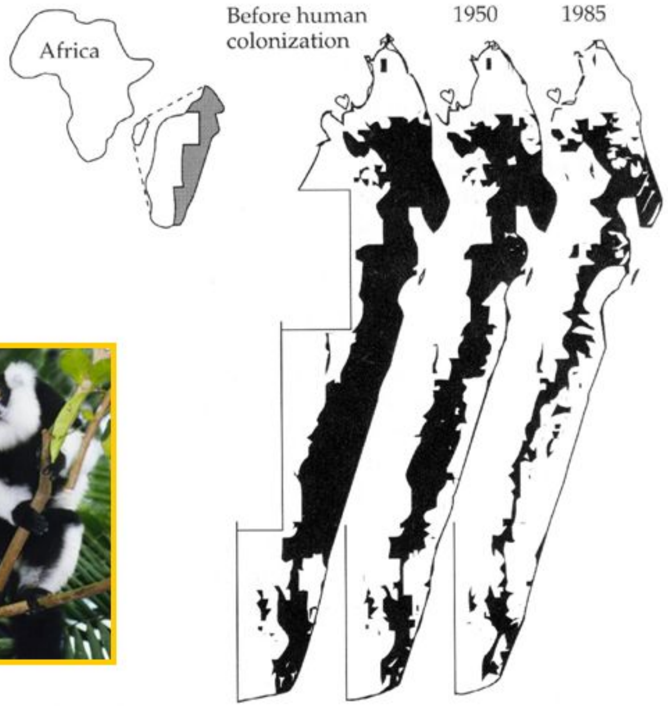
$$\frac{dp}{dt} = 0 \iff cp\,(1-p) - ep = 0$$
$$\iff p^* = 0 \text{ or } p^* = 1 - \frac{e}{c}$$

- Persistence if $p^* > 0$, i.e.

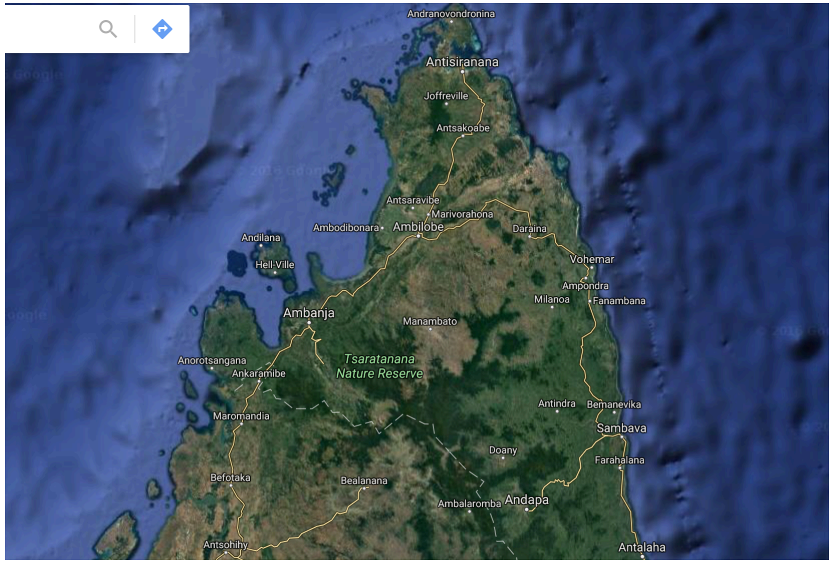
$\boldsymbol{c} > \boldsymbol{e}$ (does it make sense?)

Habitat destruction and extinction threshold

DEFORESTATION AND HABITAT FRAGMENTATION IN MADAGASCAR



Extent of eastern rainforest



Habitat destruction and equilibrium

- If we destroy the habitat, so that ***h* fraction remains**, the remaining part available for colonization is $h - p$

$$\frac{dp}{dt} = cp\,(h - p) - e\,p$$

- The new equilibrium is

$$p^* = 1 - \frac{e/c}{h}$$

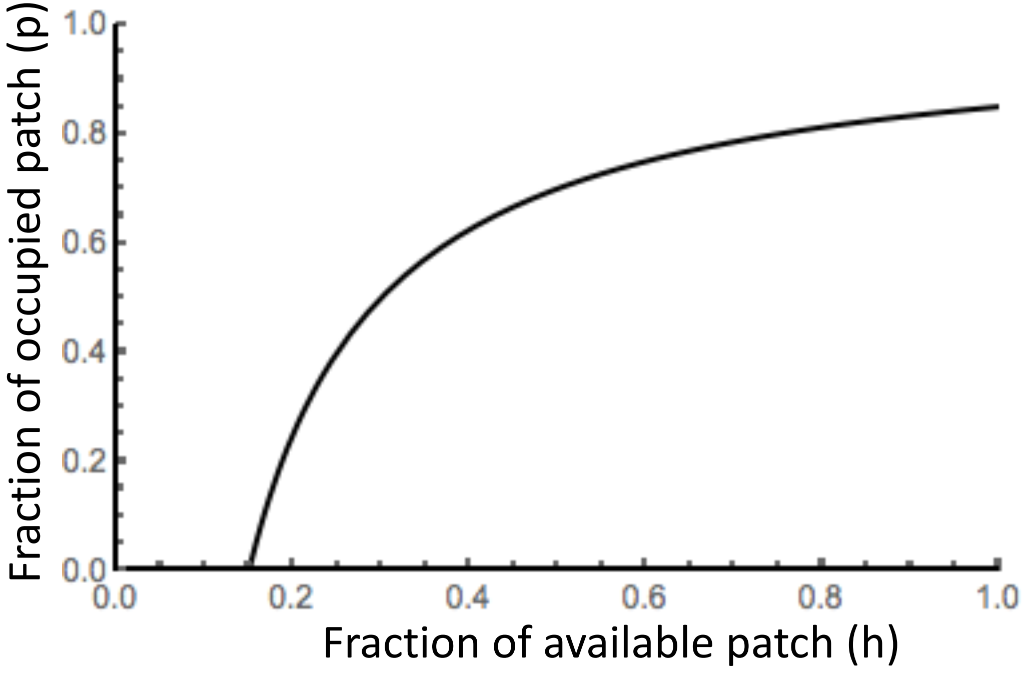
Habitat destruction and equilibrium

- If we destroy the habitat, so that ***h* fraction remains**, the remaining part available for colonization is $h - p$

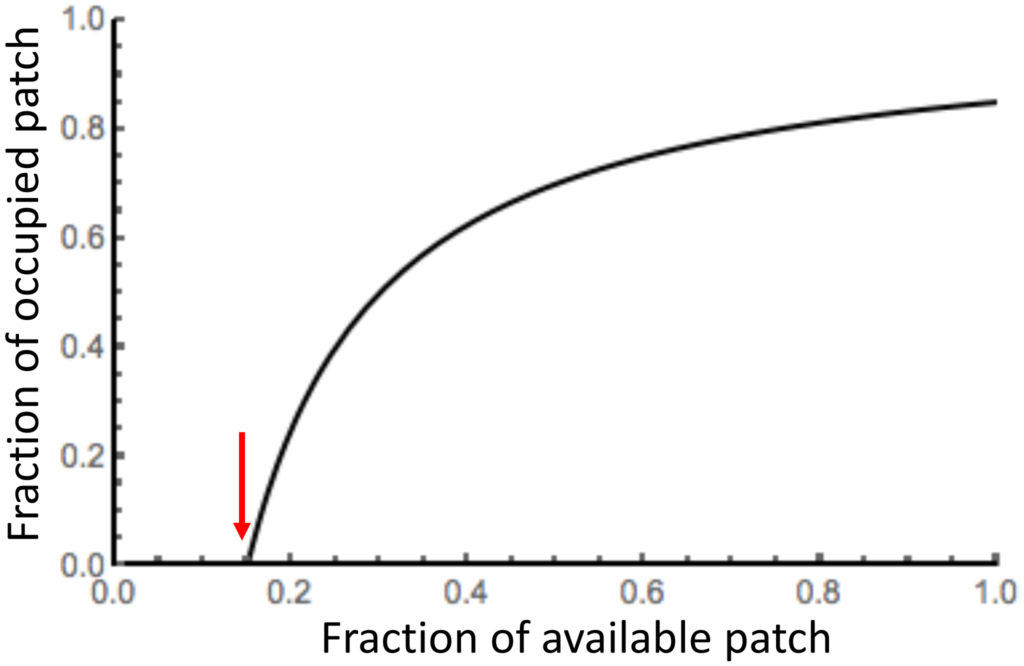
$$\frac{dp}{dt} = cp\,(h - p) - e\,p$$

- The new equilibrium is

$$p^* = 1 - \frac{e/c}{h}$$



Habitat destruction and extinction threshold



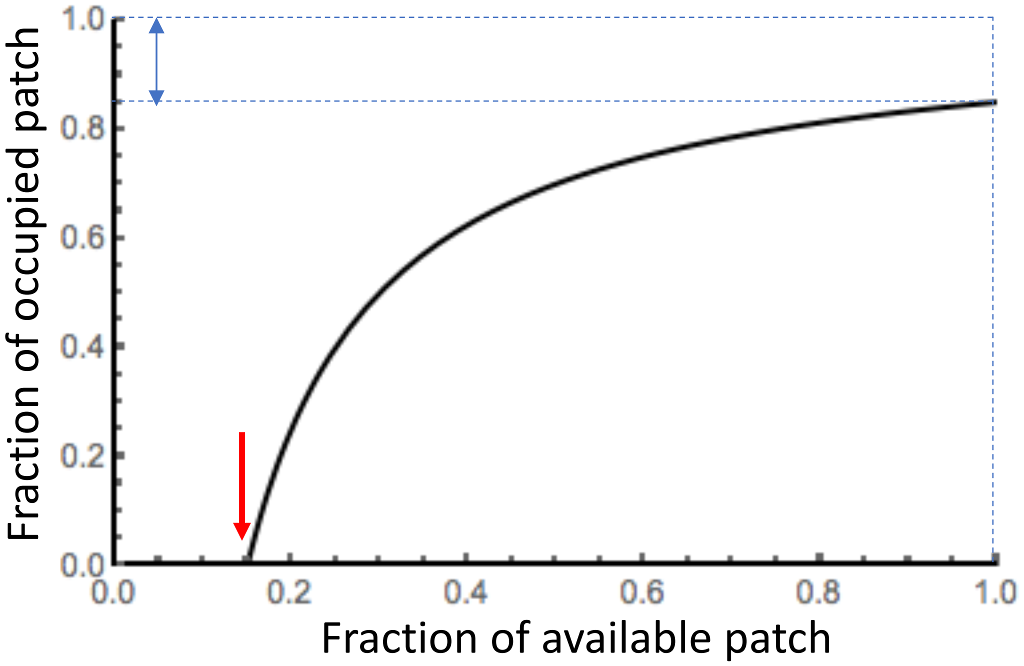
The extinction threshold (↓) is the minimum amount of habitat necessary for the persistence of the metapopulation

$$p^* = 1 - \frac{e/c}{h} > 0$$

i.e.

$$\boldsymbol{h > e/c}$$

Habitat destruction and extinction threshold



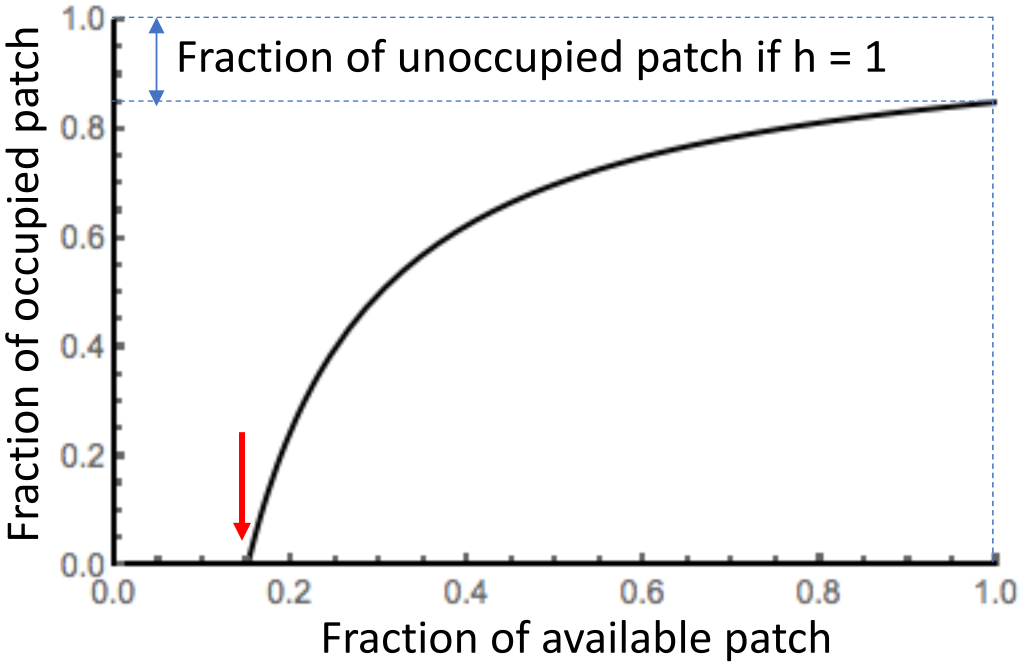
The extinction threshold (↓) is the minimum amount of habitat necessary for the persistence of the metapopulation

$$p^* = 1 - \frac{e/c}{h} > 0$$

i.e.

$$\boldsymbol{h > e/c}$$

Habitat destruction and extinction threshold



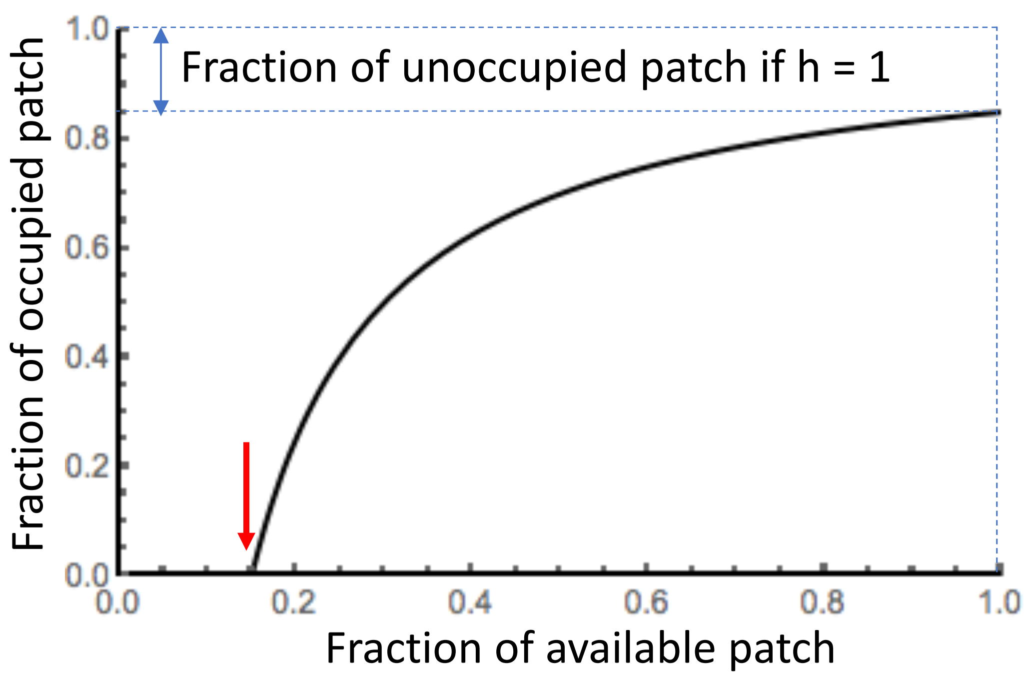
The extinction threshold (↓) is the minimum amount of habitat necessary for the persistence of the metapopulation

$$p^* = 1 - \frac{e/c}{h} > 0$$

i.e.

$$\boldsymbol{h > e/c}$$

Habitat destruction and extinction threshold



The extinction threshold (↓) is the minimum amount of habitat necessary for the persistence of the metapopulation

$$p^* = 1 - \frac{e/c}{h} > 0$$

i.e.

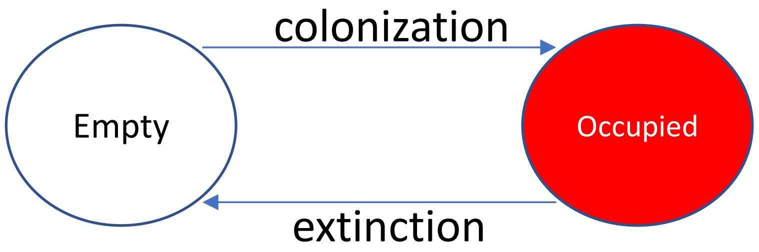
$$\boldsymbol{h > e/c}$$

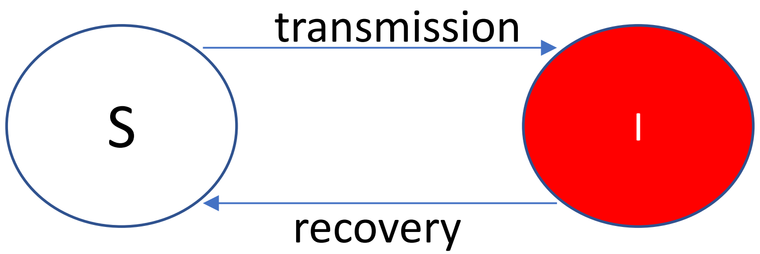
The extinction threshold = the fraction of unoccupied patch at equilibrium when all patches are available

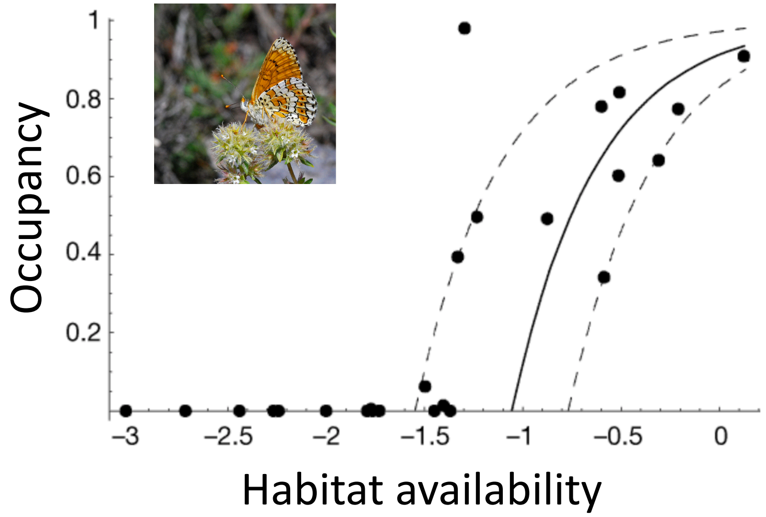
Ecology vs. Epidemiology

Ecology ~ Epidemiology

Metapopulation ~ Epidemiology: SIS

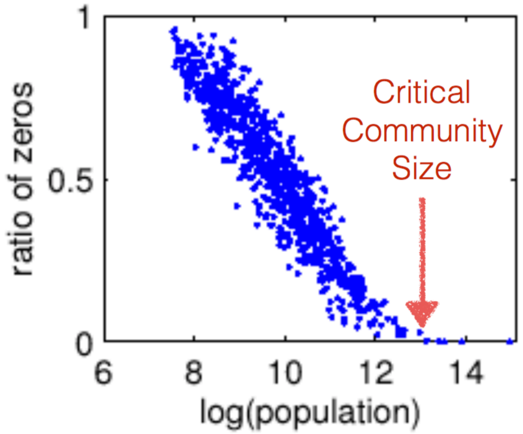






Hanski & Ovaskainen, 2000, *Nature*

Smaller cities go extinct more often



Grenfell et al., 2001, *Nature*

Metapopulation ~ Epidemiology: SIS

Metapopulation

- The fraction of occupied patch at equilibrium is

$$p^* = 1 - \frac{\text{extinction rate}}{\text{colonization rate}} = 1 - \frac{e}{c}$$

- If *h* is the fraction of suitable patches, the metapopulation goes extinct if

$$h < \frac{e}{c}$$

SIS

- The fraction of infected individuals at equilibrium is

$$p^* = 1 - \frac{\text{recovery rate}}{\text{transmission rate}} = 1 - \frac{\gamma}{\beta}$$

- If *h* = 1- p_c is the fraction of unvaccinated individual, the disease is eradicated if

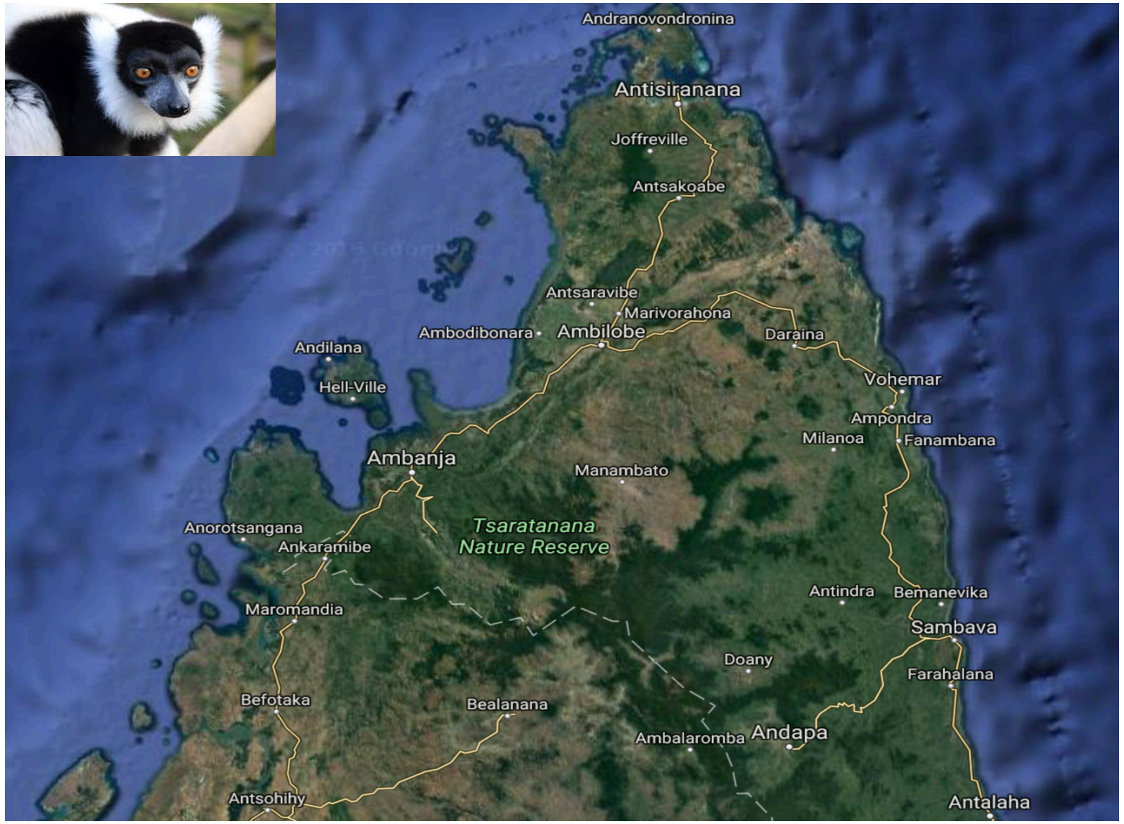
$$h < \frac{\gamma}{\beta} = \frac{1}{R_0}$$

Outline

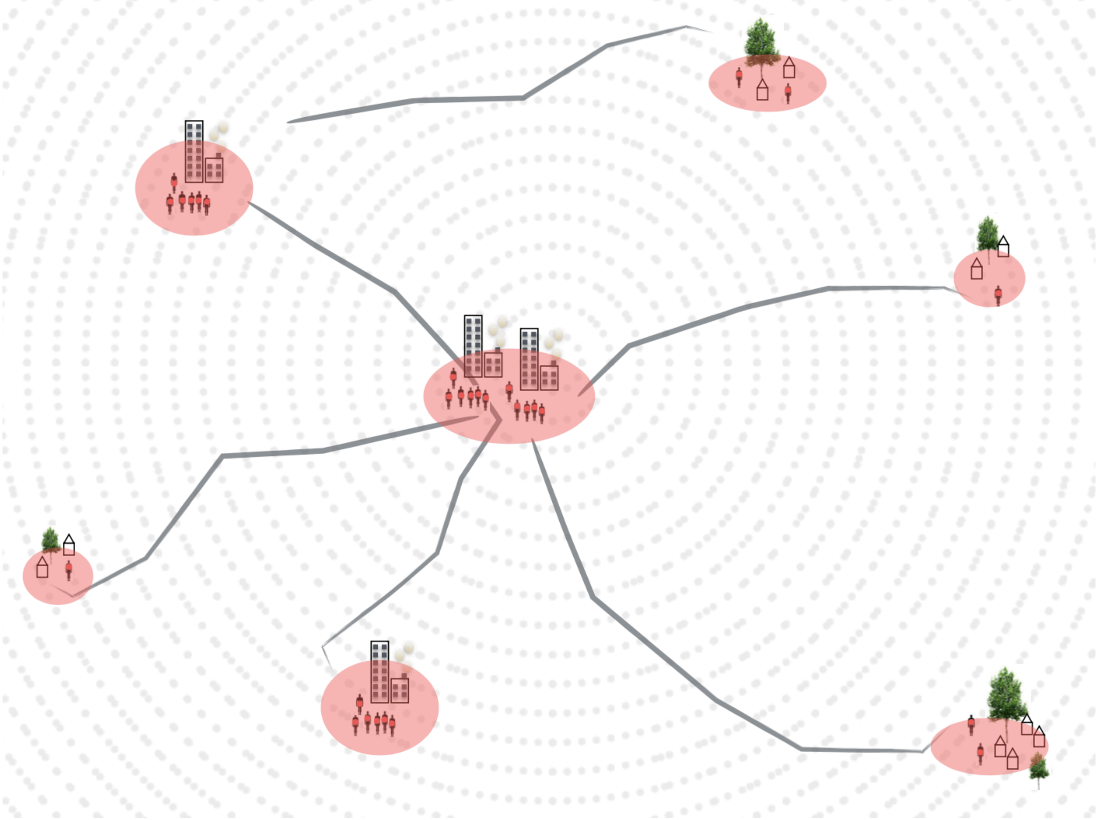
- Metapopulation paradigm
- Levins’ (spatially implicit) metapopulation model
- Spatially realistic metapopulation model
- Software demonstration: SPOMSIM

Spatially realistic metapopulation model

Ecology

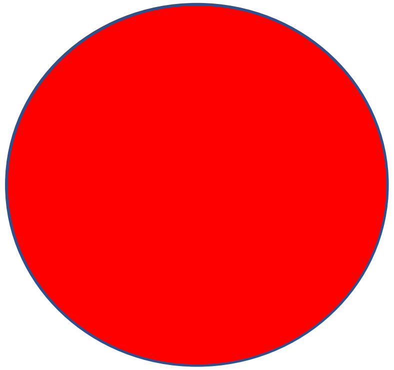


Epidemiology



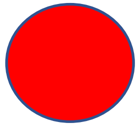
Extinction as a function of patch size

Large patch => large population size



=> Low probability of extinction

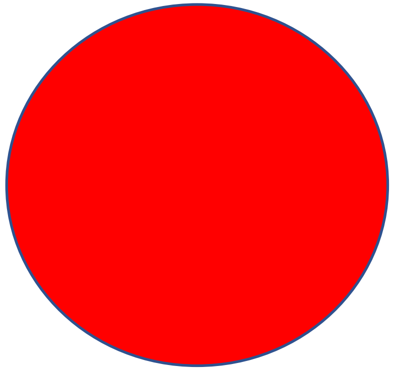
Small patch => small population size



=>High probability of extinction

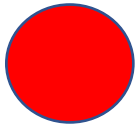
Recovery as a function of human 'condition'

Strong person => …



=> Quick recovery

Weak person => …

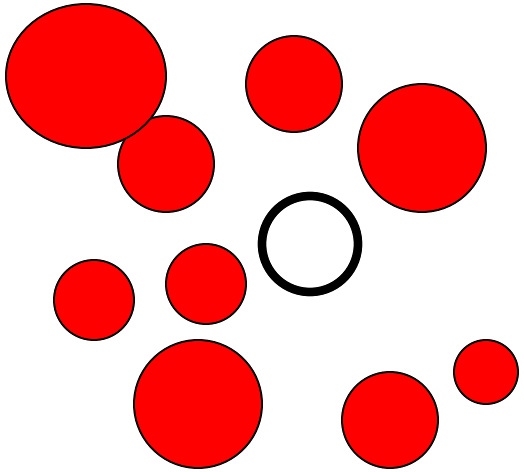


=> Slow recovery

Ecology

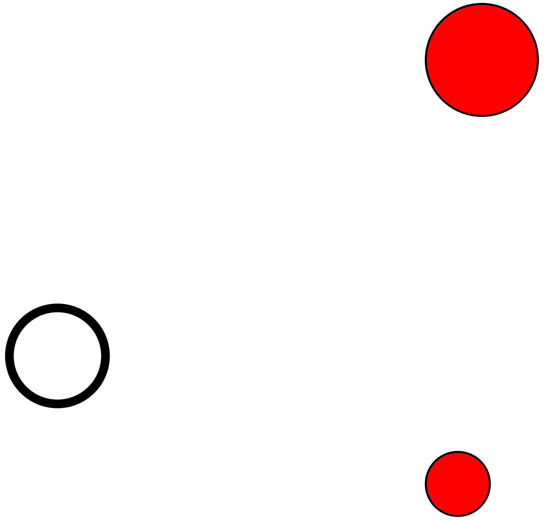
Colonization of an empty patch:
general idea = scales with *connectivity*. Heuristically:

Well-connected focal patch
(many close-by occupied neighbours)



$\Rightarrow$ **High** probability of being colonized

Poorly connected focal patch



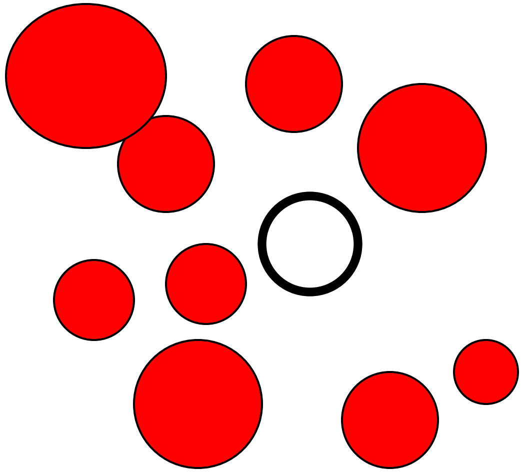
$\Rightarrow$ **Low** probability of being colonized

Epidemiology

Infection of an healthy person:
general idea = scales with *connectivity*. Heuristically:

Well-connected focal healthy person

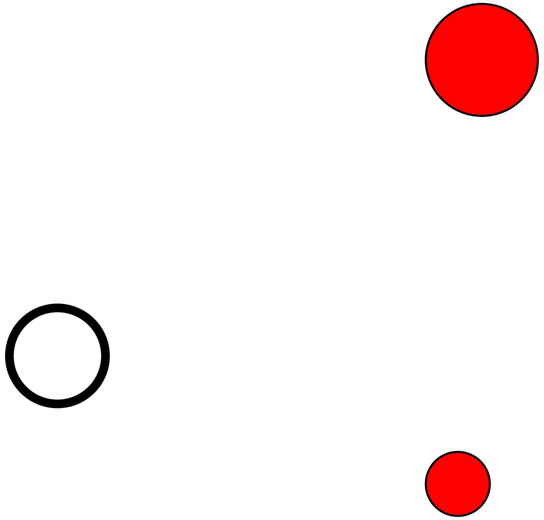
(many sick persons close-by)



⇒ **High** probability of being infected

Poorly connected

focal health person



⇒ **Low** probability of being infected

A little bit of math: functional forms

Extinction (recovery) probability

- Assume patch *i* of size A_i is occupied, the probability of local extinction is

$$prob_e(A_i) = 1 - e^{-e/A_i}$$

A little bit of math: functional forms

Extinction (recovery) probability

- Assume patch *i* of size A_i is occupied, the probability of local extinction is

$$prob_e(A_i) = 1 - e^{-e/A_i}$$

Colonization (transmission) probability

- Assume patch *i* of size A_i is empty, the probability of local colonization is

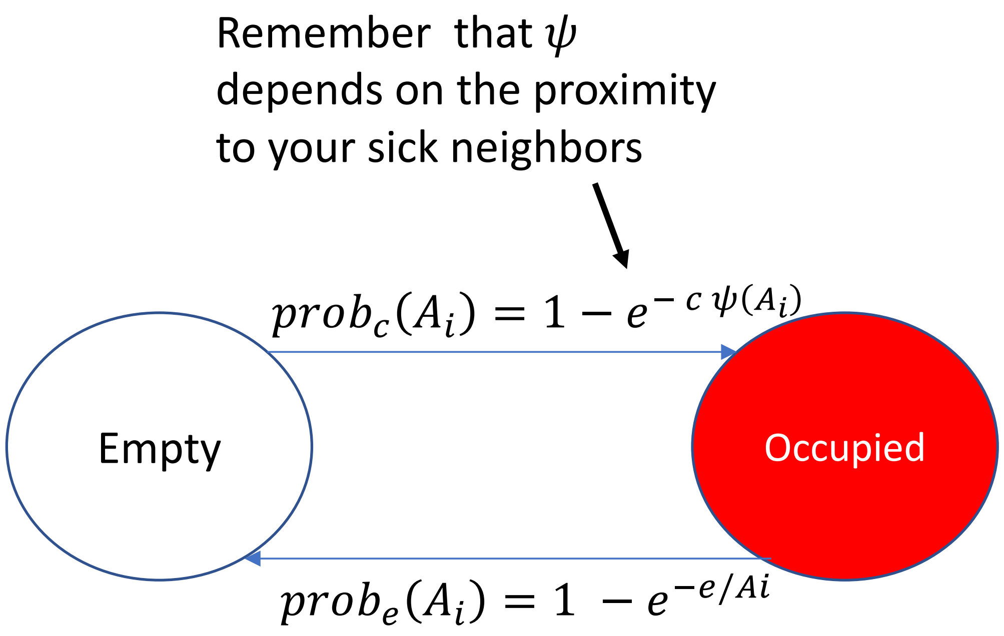
$$prob_c(A_i) = 1 - e^{-c\,\psi(A_i)}$$

- $\psi(A_i)$ represents the connectivity of the patch *i*

$$\psi(A_i) = A_i \sum_{A_j \in\, source} A_j\, e^{-\kappa\, d_{ij}}$$

- $e^{-\kappa\, d_{ij}}$ is called a dispersal kernel

Simulation



'for' loop for each time step

- If patch 1 is empty, it has a probability $prob_c(A_1)$ to be colonized for next year

- If patch 1 is occupied, it has a probability $prob_e(A_1)$ to go extinct for next year

Repeat that procedure for all the patches in the system and we get the next state of the metapopulation

Extinction threshold in the realistic model

- In the Levins' model, the equilibrium was

$$p^* = 1 - \frac{e/c}{h}$$

Extinction threshold in the realistic model

- In the Levins' model, the equilibrium was

$$p^* = 1 - \frac{e/c}{h}$$

- In the realistic model, we have a similar relationship

$$p^* = 1 - \frac{e/c}{\lambda_M}$$

- λ_M summarizes the role of dispersal, landscape structure, and the dominant eigenvalue of a something called "landscape" matrix.
- λ_M is called the **metapopulation capacity**

Hanski & Ovaskainen 2000

Outline

- Metapopulation paradigm
- Levins’ (spatially implicit) metapopulation model
- Spatially realistic metapopulation model
- Software demonstration: SPOMSIM

SPOMSIM: Stochastic Patch Occupancy Model Simulation

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Ecological Modelling 179 (2004) 533–550

ECOLOGICAL MODELLING

www.elsevier.com/locate/ecolmodel

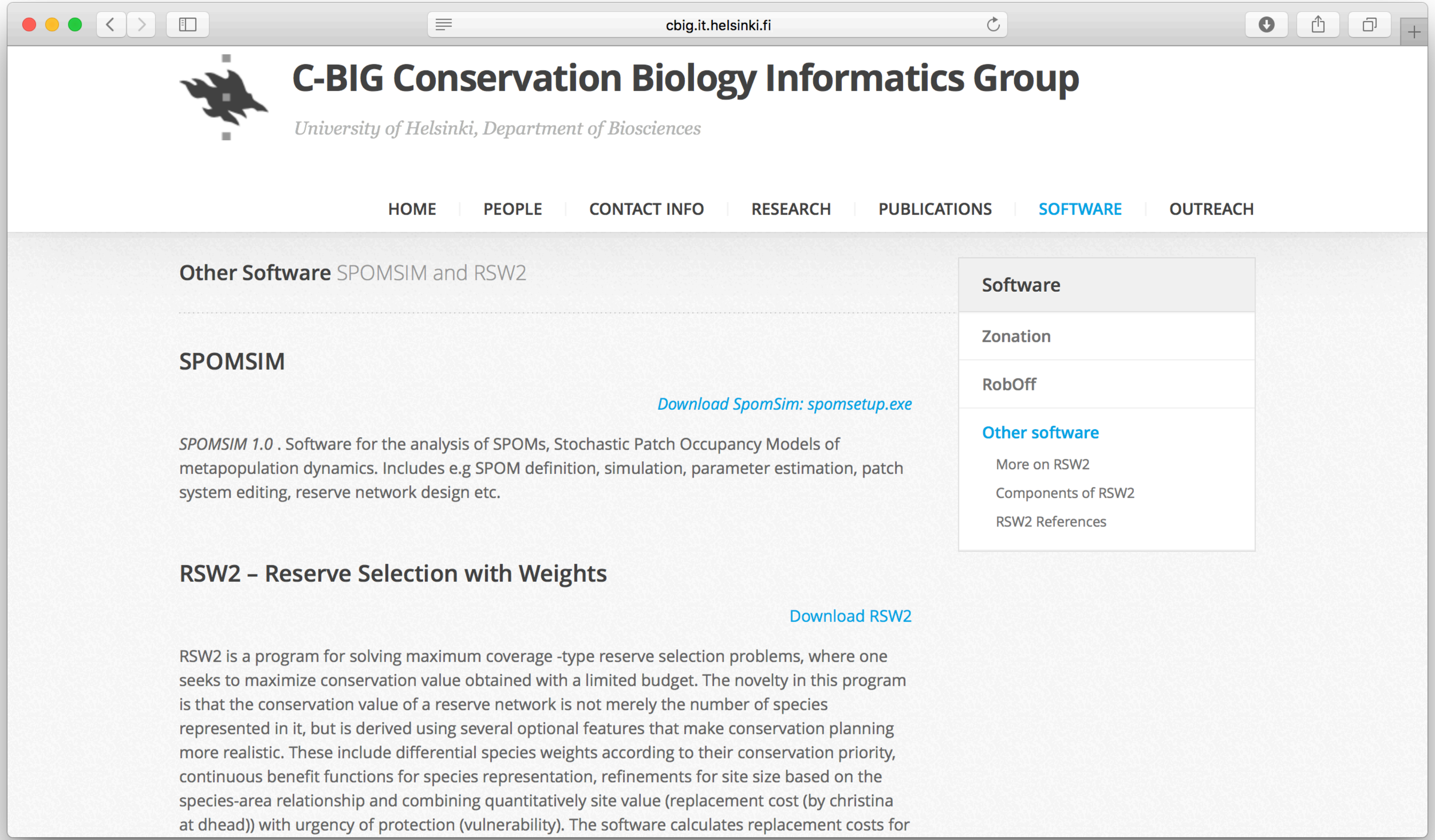
SPOMSIM: software for stochastic patch occupancy models of metapopulation dynamics

Atte Moilanen*

Department of Biological and Environmental Sciences, Metapopulation Research Group, University of Helsinki, PO Box 65 (Viikinkaari 1), Helsinki FIN-00014, Finland

Received 7 July 2003; received in revised form 29 March 2004; accepted 29 April 2004

SPOMSIM: Stochastic Patch Occupancy Model Simulation



Conclusions

Spatial models are needed when:

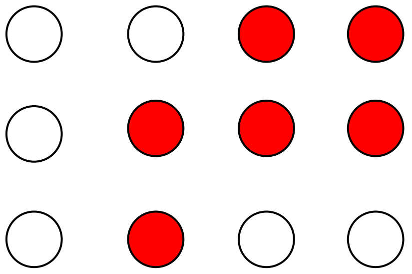
- the question involves both the distribution and abundance of a species/disease
- When dispersal and landscape structure are important processes

Conclusions

Spatial models are needed when:

- the question involves both the distribution and abundance of a species/disease
- When dispersal and landscape structure are important processes

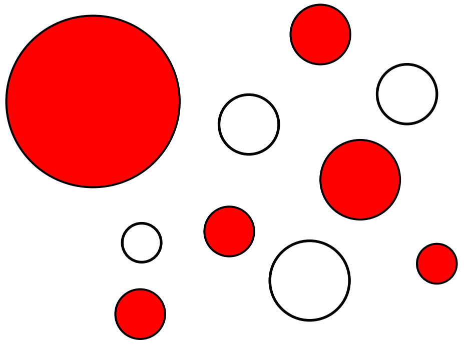
Spatially implicit: homogenous



$$\text{Habitat quality} > \frac{\text{extinction rate}}{\text{colonization rate}}$$

$$\text{Susceptible} > \frac{\text{recovery rate}}{\text{transmission rate}}$$

Spatially realistic: heterogeneous

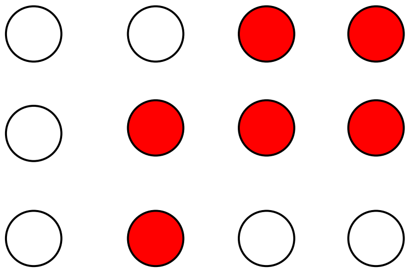


Conclusions

Spatial models are needed when:

- the question involves both the distribution and abundance of a species/disease
- When dispersal and landscape structure are important processes

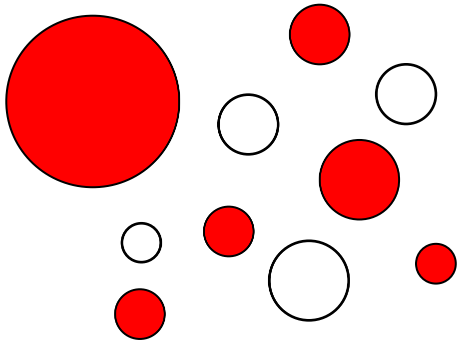
Spatially implicit: homogenous



$$\text{Habitat quality} > \frac{\text{extinction rate}}{\text{colonization rate}}$$

$$\text{Susceptible} > \frac{\text{recovery rate}}{\text{transmission rate}}$$

Spatially realistic: heterogeneous



Metapopulation theory helps to understand dynamics of population in a fragmented landscape