Introduction to Compartmental Models

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- Understand the difference between statistical and mechanistic models
  Comprendre la différence entre les modèles statistiques et mécanistes.

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  Comprendre comment on peut formuler et conceptualiser les modèles compartimentés

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- Example: population growth, predator prey, SIR models

?/?/? Models

Compartmental/Mechanistic/Mathematical/ Dynamical Models

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   Les populations sont subdivisées en compartiments

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   Les compartiments et les taux de transition sont déterminés par les systèmes biologiques

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3. Compartments and transition rates are determined by biological systems
   Les compartiments et les taux de transition sont déterminés par les systèmes biologiques
4. Rates of transferring between compartments are expressed mathematically
   Taux de transition entre les compartiments sont exprimés mathématiquement

How are these different from statistical models?

En quoi sont-ils différents des modèles statistiques?

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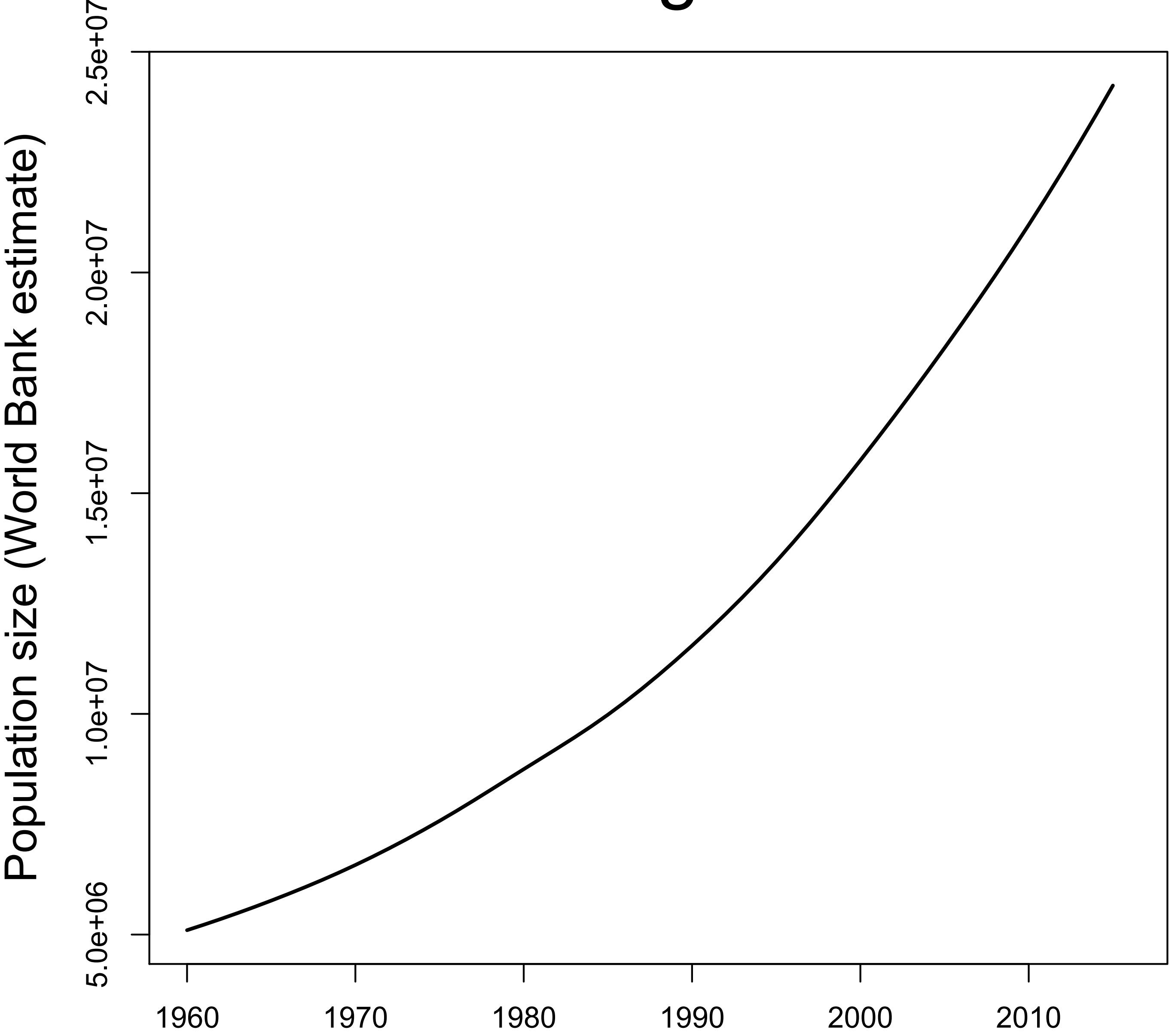
En quoi sont-ils différents des modèles statistiques?

Make explicit hypotheses about biological mechanisms that drive dynamics (may not be realistic, but still explicit)

Faire des hypothèses explicites sur les mécanismes biologiques qui régissent la dynamique de l'infection (peut ne pas être réaliste, mais toujours explicite)

1. Simple Population Models
1. Modèles simples de population

Madagascar



http://databank.worldbank.org

What can we say about the population of Madagascar?

How would a model help us? What kind of model should we use?

The basic population model

Compartmental models (Mechanistic Models)

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How does the population of Madagascar grow over time?

Comment est-ce que la population de Madagascar s’augmente avec le passage du temps?

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Square = compartment

N = state variable = the data we want to explain

What is the ***precise*** definition of N?

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Madagascar (N)

The basic population model

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Madagascar
(N)

How does the population grow?

The basic population model

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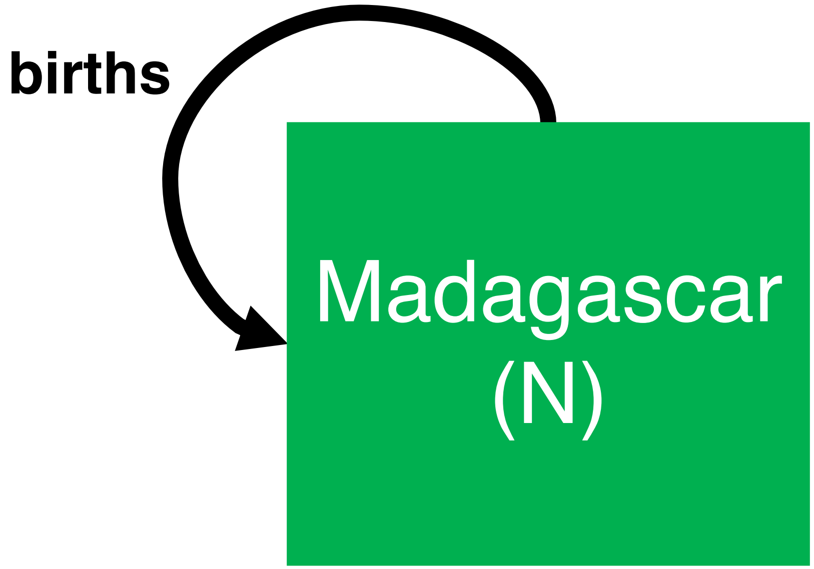

How does the population decrease?

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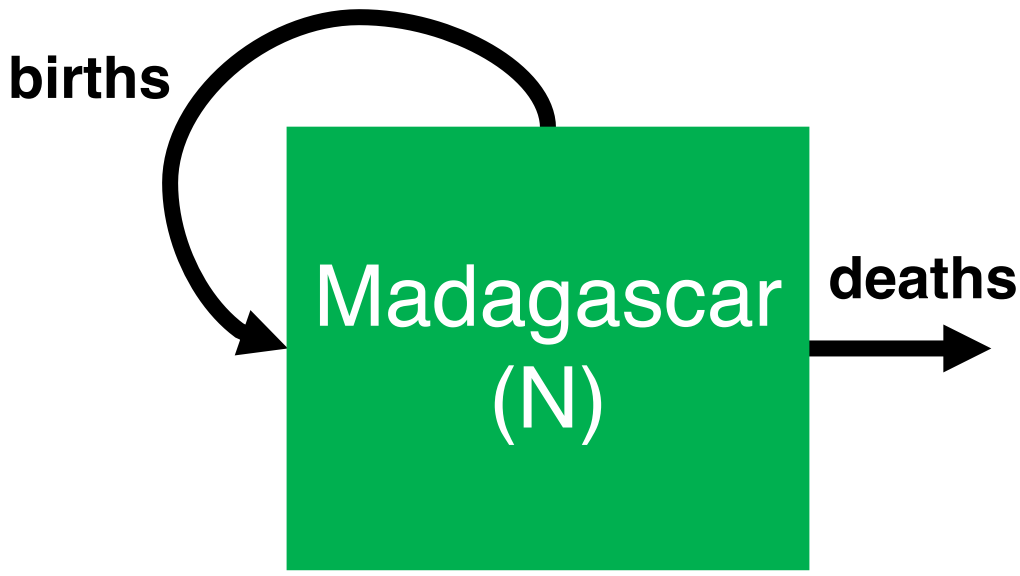

What is a big assumption we are making here?

The basic population model

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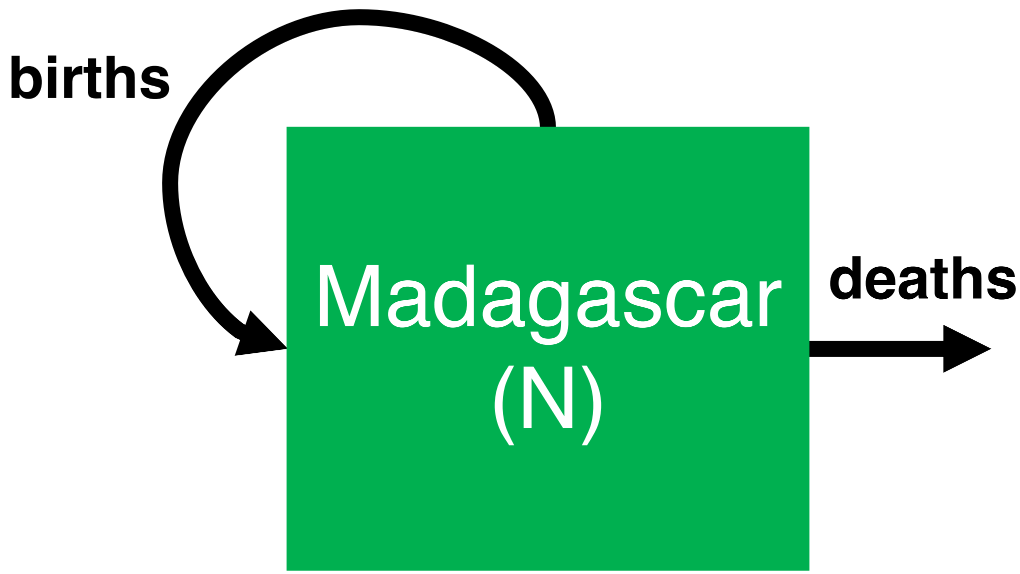

$N_{t+1} =$

The basic population model

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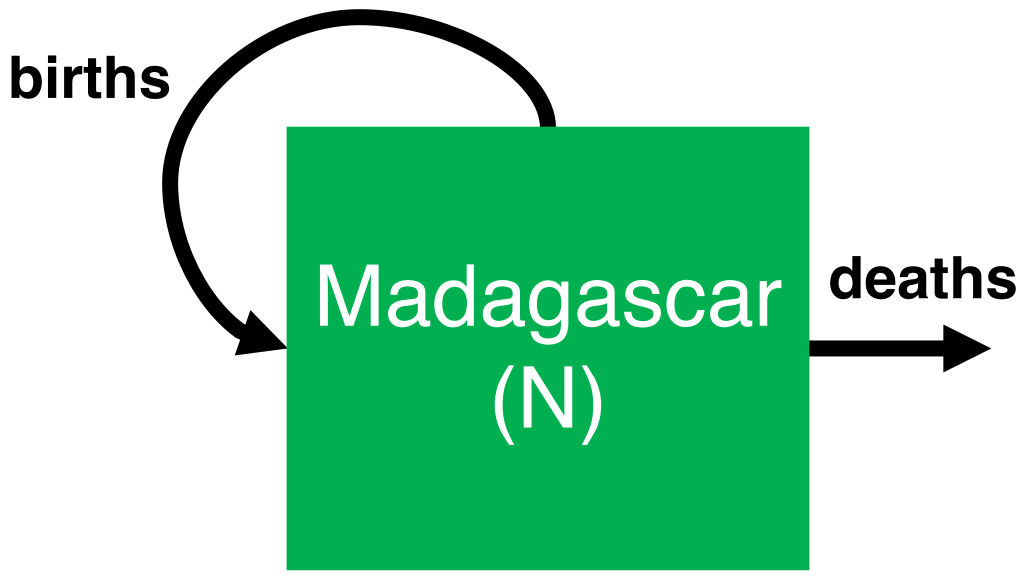

$$N_{t+1} = births * N_t$$

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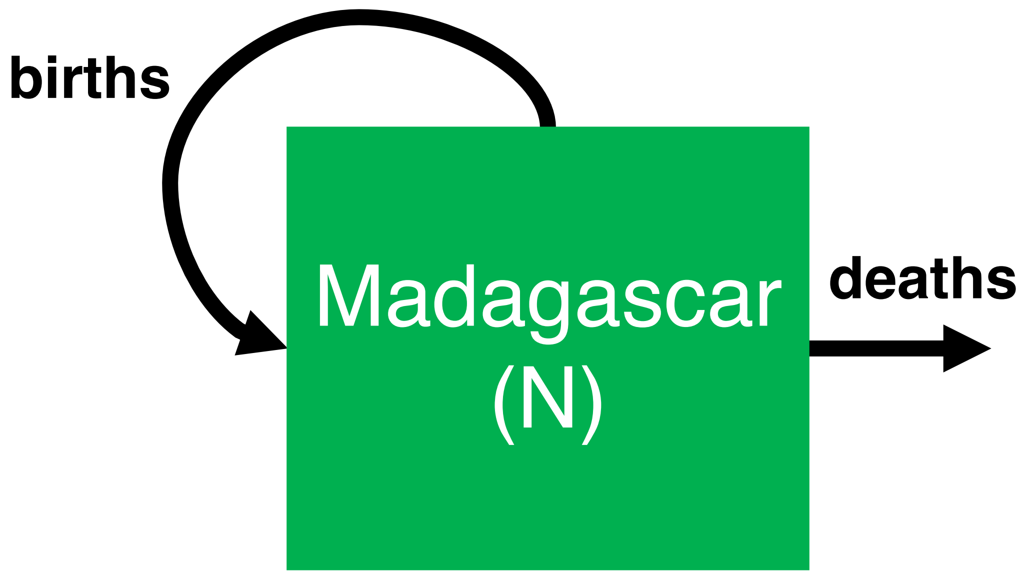

$$N_{t+1} = births * N_t - deaths * N_t$$

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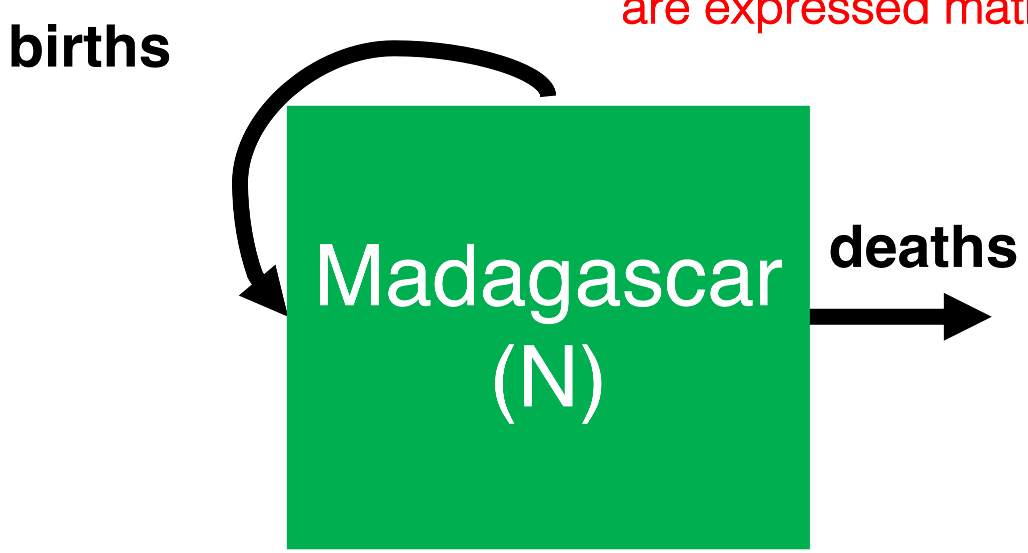


$$N_{t+1}=births*N_t-deaths*N_t$$

$$N_{t+1}=(births-deaths)*N_t$$

$$N_{t+1}=\lambda*N_t$$

λ=pop intrinsic growth rate

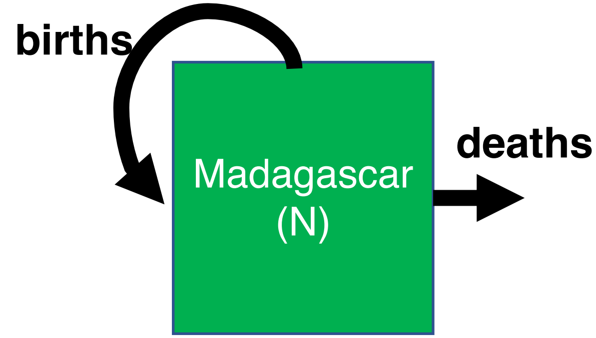
What are the main assumptions of a simple population model?

Closed population
Homogenous mixing
Same birth and death rate for each person

What is lambda?

Population intrinsic growth rate

The basic population model



Population rate of increase

Taux d'accroissement de la population

$$\lambda = \frac{N_t}{N_{t+1}}$$

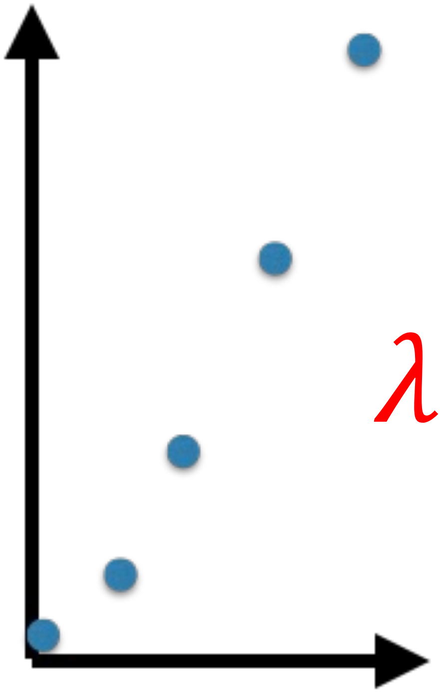
pop size at t

pop size at t+1

This is for one time step, how do we generalize this equation to work for all time steps?

The basic population model

Discrete time



$$\lambda = \frac{N_t}{N_{t+1}}$$

$N_1 =$

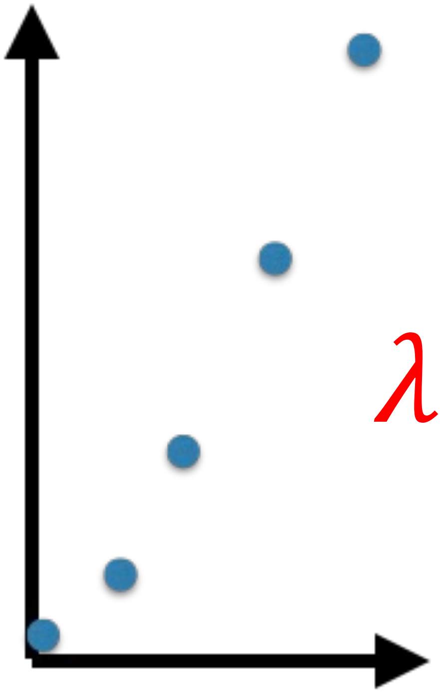
$N_2 =$

$N_3 =$

$N_t =$

The basic population model

Discrete time



$$\lambda = \frac{N_t}{N_{t+1}}$$

$N_1 = N_0\lambda$

$N_2 =$

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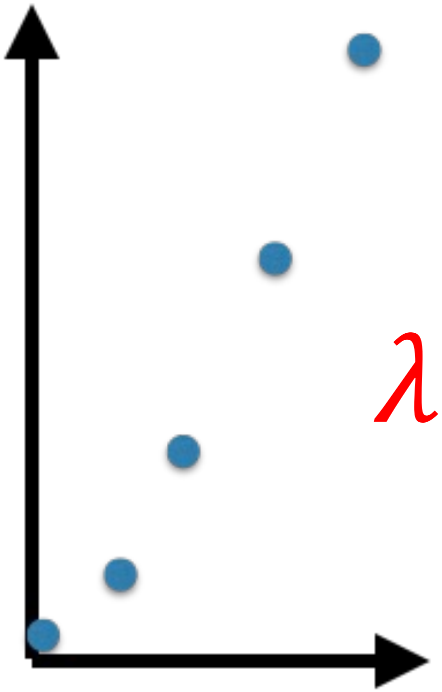

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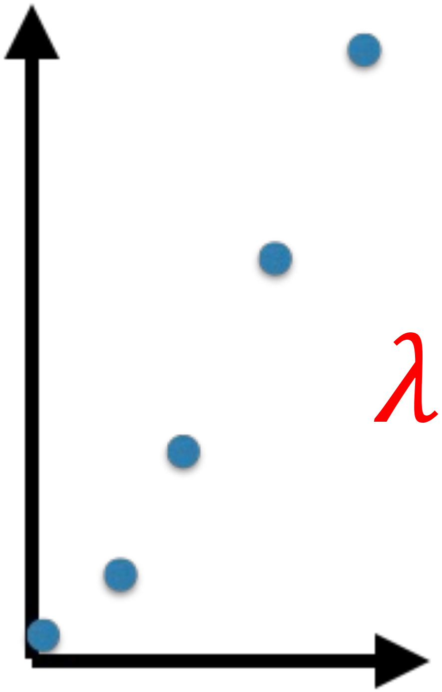
$$N_2 = [N_0\lambda]\lambda = \lambda^2 N_0$$

$$N_3 =$$

$$N_t =$$

The basic population model

Discrete time



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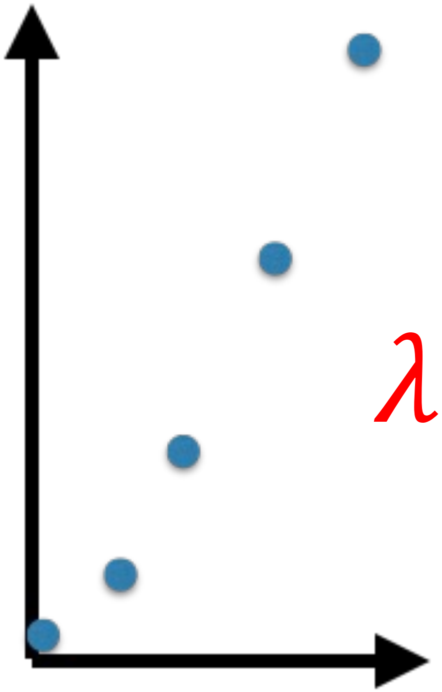

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$$N_3 = \lambda^3 N_0$$
$$N_t = \lambda^t N_0$$

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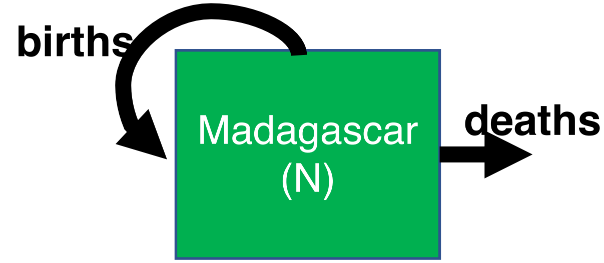
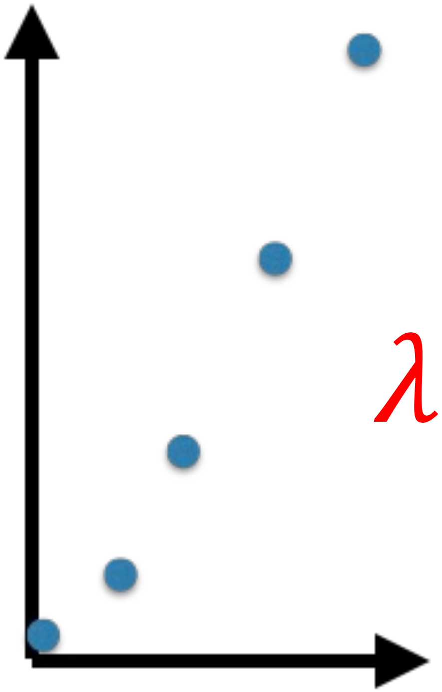


Discrete time

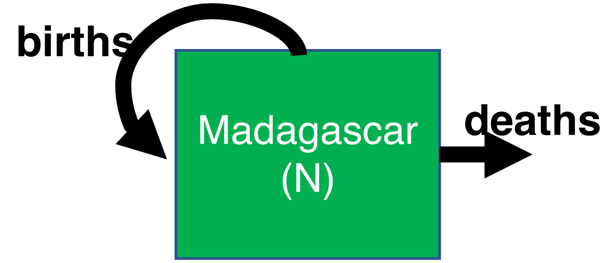


$$\lambda = \frac{N_t}{N_{t+1}}$$

What if we want to know the population size for any time t? Not just where we have data?

$$N_t = \lambda^t N_0$$

The basic population model



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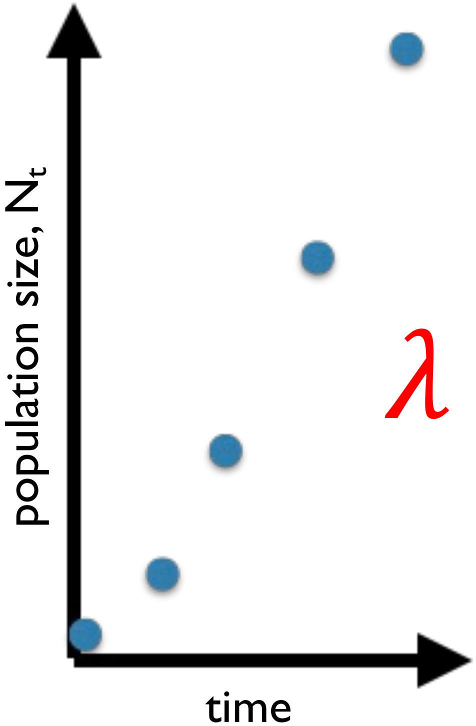
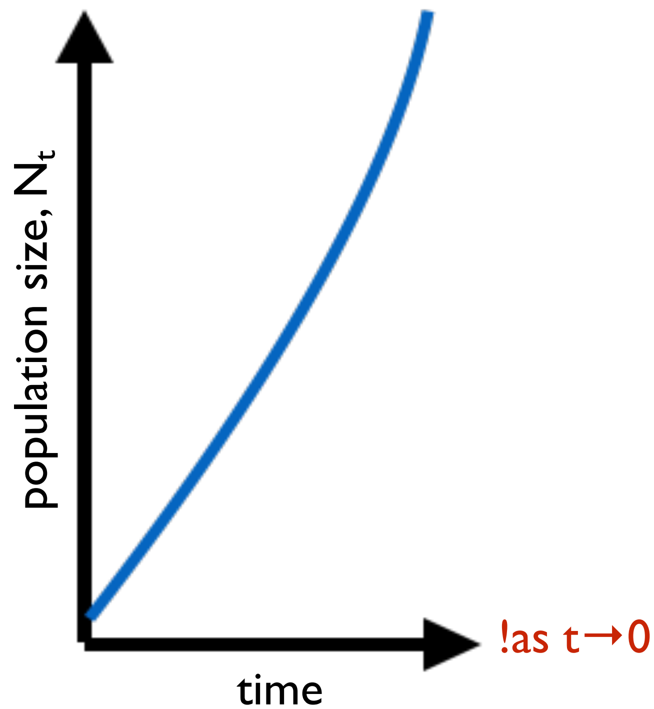


$$\lambda = \frac{N_t}{N_{t+1}}$$

$$N_t = \lambda^t N_0$$

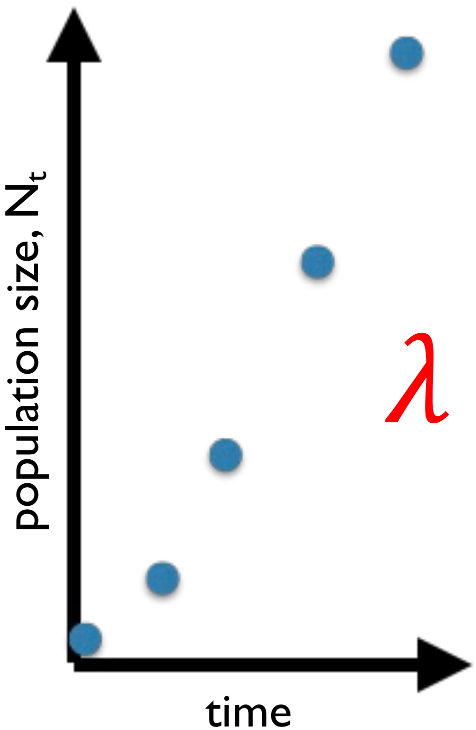
Continuous time



How do we get the same type of equation but for continuous time?

The basic population model

Discrete time



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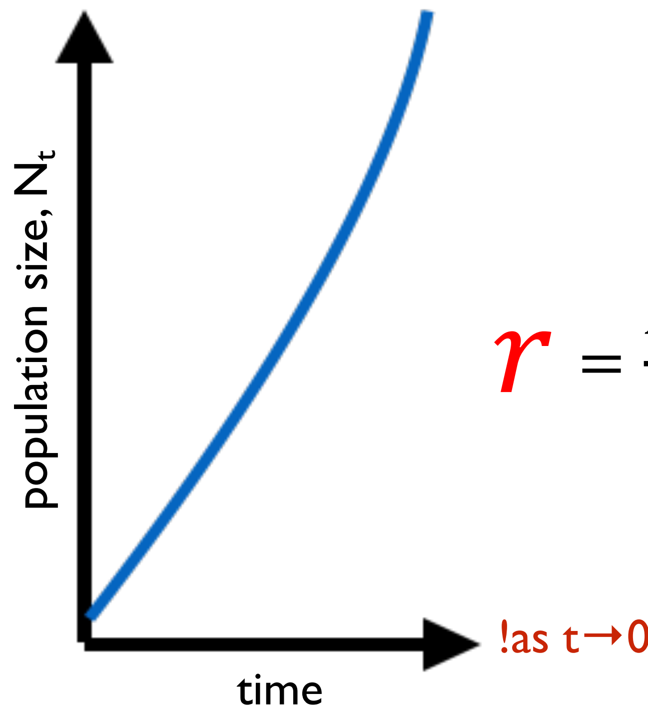
Continuous time



$$r = \frac{N_t - N_0}{t}$$

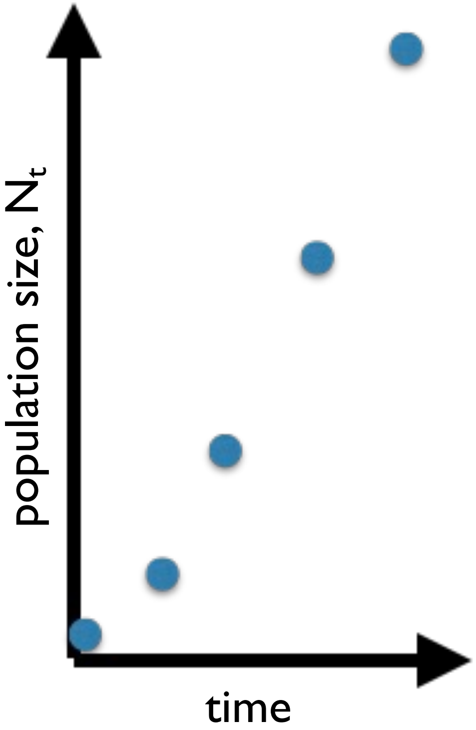
$$\frac{dN}{dt} = rN$$

The basic population model

Continuous time

$$\frac{dN_t}{dt} = rN_t$$

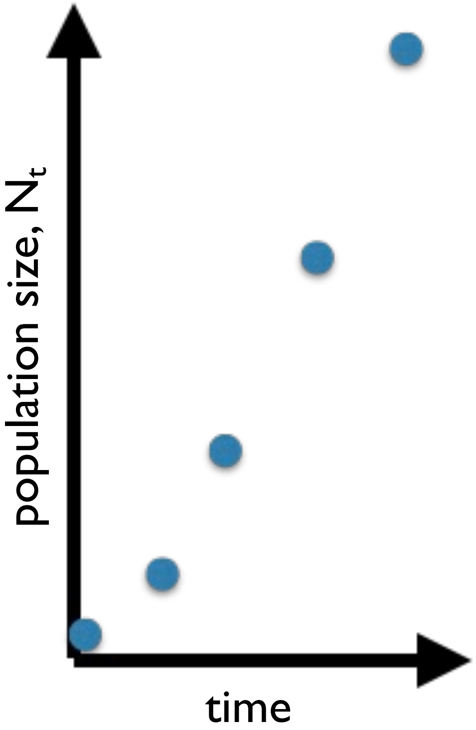
Discrete time



$$N_t = \lambda^t N_0$$

The basic population model

Discrete time



$$N_t = \lambda^t N_0$$

Continuous time

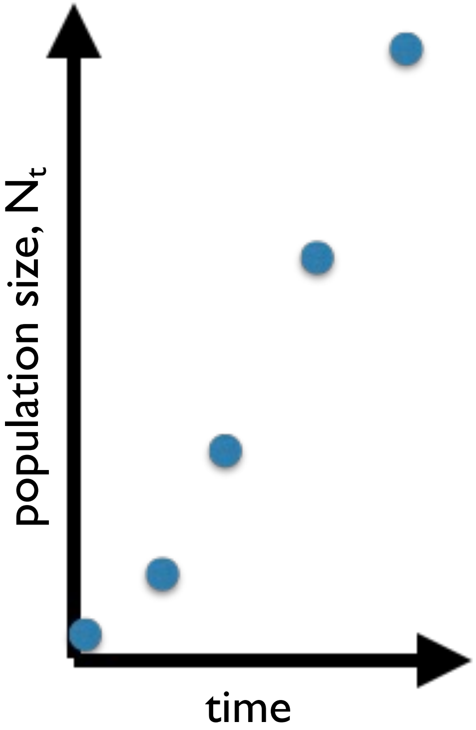
$$\frac{dN_t}{dt} = rN_t$$

Separation of variables:

$$\frac{dN_t}{N_t} = r\,dt$$

The basic population model

Discrete time



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Continuous time

$$\frac{dN_t}{dt} = rN_t$$

Separation of variables:

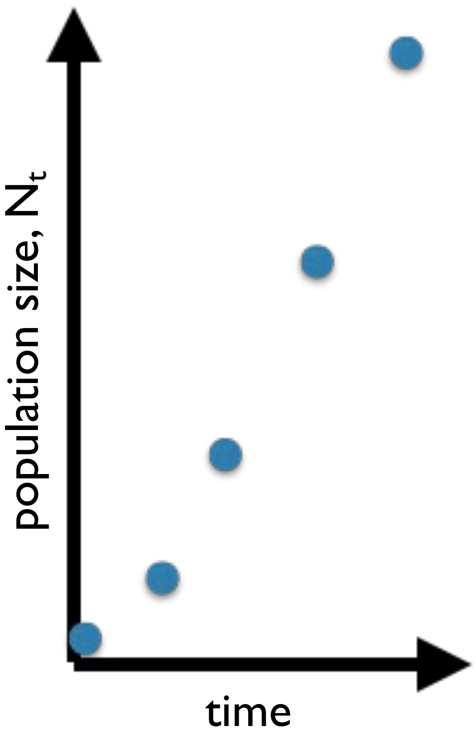
$$\frac{dN_t}{N_t} = r\,dt$$

Integrate both sides:

$$\int_{N_0}^{N_t} \frac{dN_t}{N_t} = r\int_0^t dt$$

The basic population model

Discrete time



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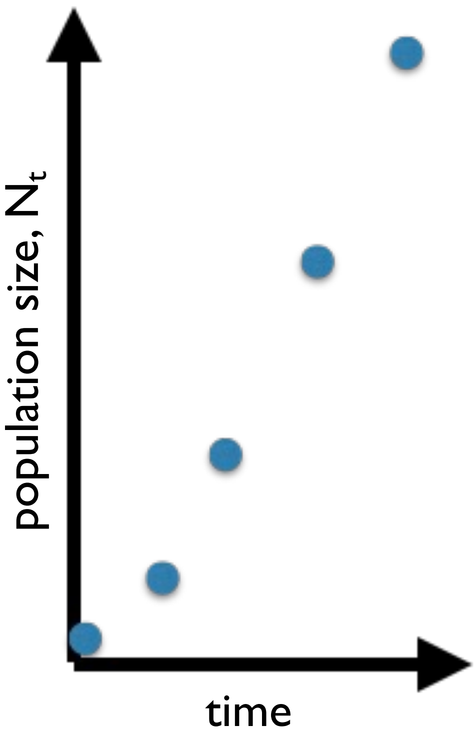
By definition:

$$\ln N_t - \ln N_0 = rt$$

$$\ln\left(\frac{N_t}{N_0}\right) = rt$$

The basic population model

Discrete time



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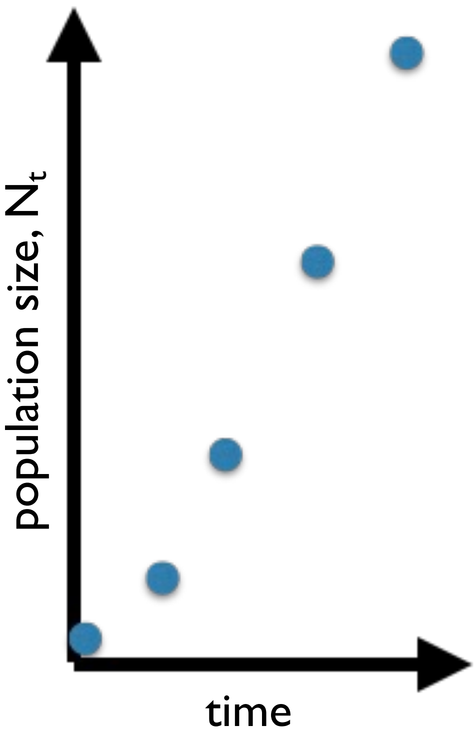
Take exponentials:

$$\frac{N_t}{N_0} = e^{rt}$$

The basic population model

Discrete time



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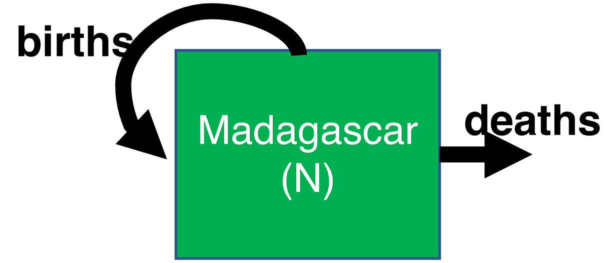
Take exponentials:

$$\frac{N_t}{N_0} = e^{rt}$$

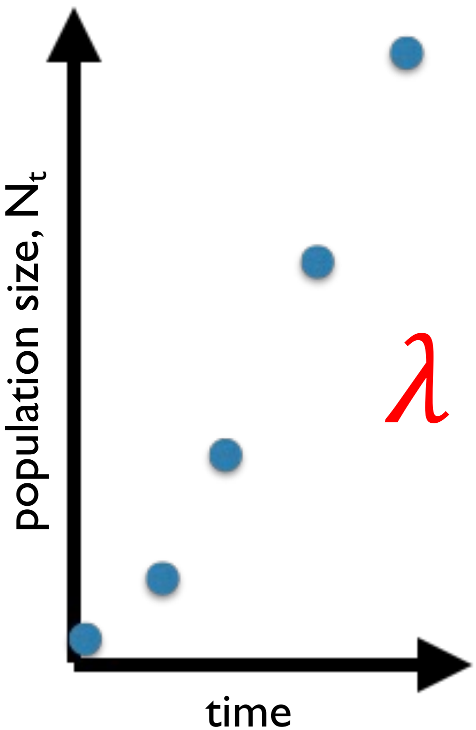
Solve for N(t):

$$N_t = N_0 e^{rt}$$

The basic population model

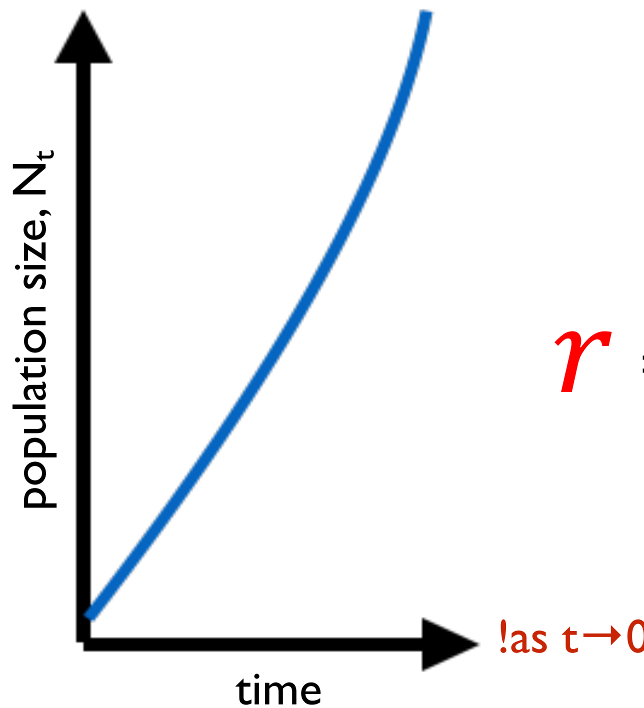


Discrete time



$$\lambda = \frac{N_t}{N_{t+1}}$$

Continuous time



$$r = \frac{N_t - N_0}{t}$$

$$N_t = \lambda^t N_0$$

$$N_t = N_0 e^{rt}$$

The basic population model

Discrete

Continuous

Continuous models can be discretized; discrete models can be approximated by continuous ones. The appropriate framing may depend on the data / question.

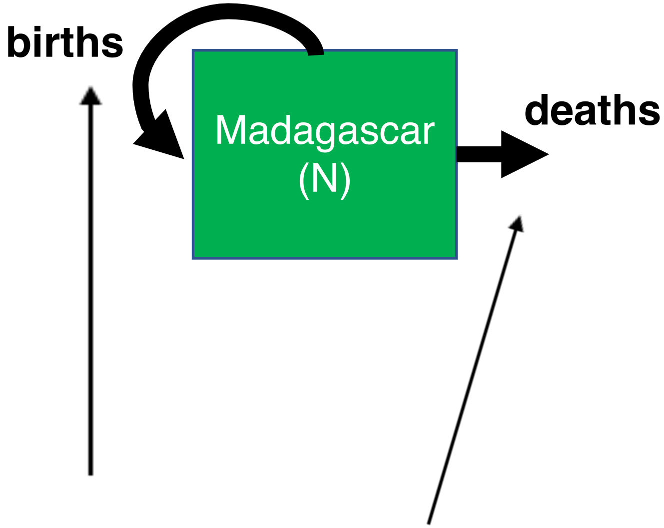
What is the difference between discrete and continuous models?

Discrete: state variable only changes at distinct time steps
Continuous: state variables change continuously (tiny tiny time steps)

What math is used in discrete pop models? Continuous pop models?

Algebra, calculus

The basic population model

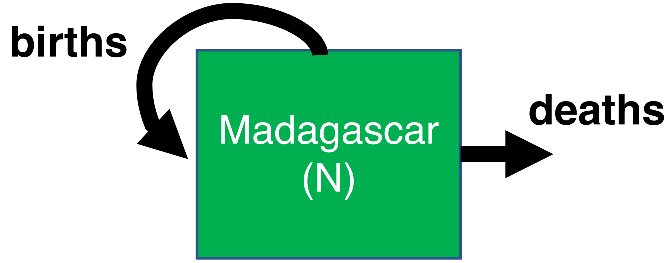


What about those rates?
Are they the same every year?
And in every person? Why might they be different?

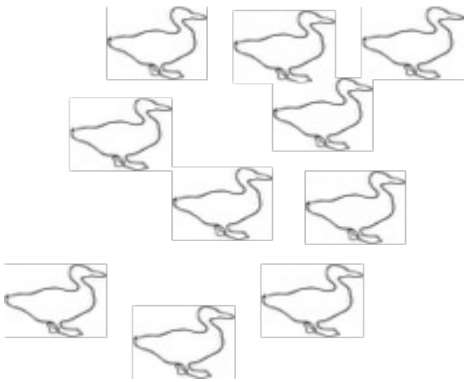
Reproductive age
Death rate increasing with age
Diseases/other health factors

How do we incorporate this ‘randomness’?

The basic population model



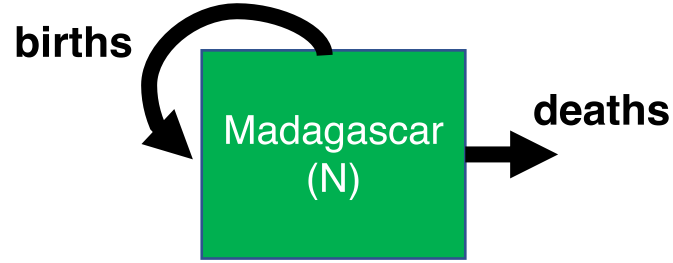
starting population



probability of death = 0.5

if deterministic "always the same"

The basic population model



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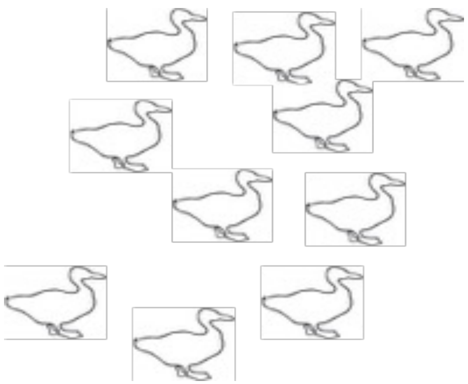

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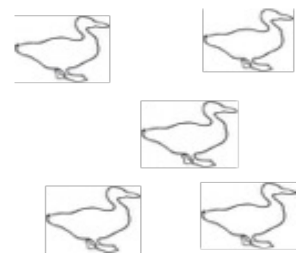
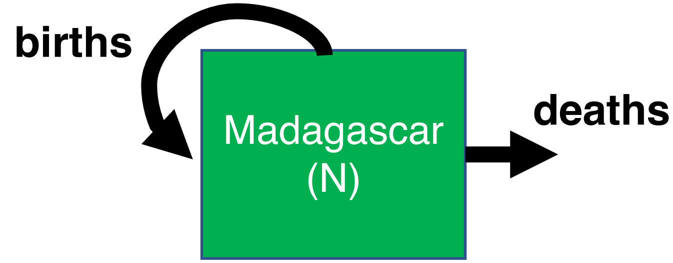


The basic population model



starting population

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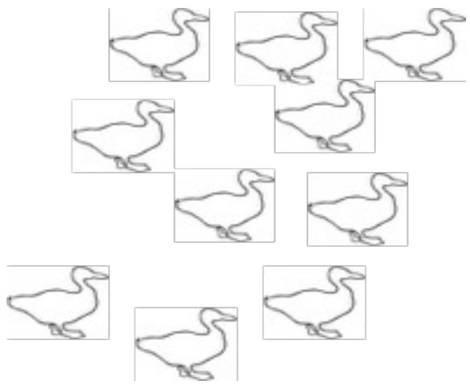

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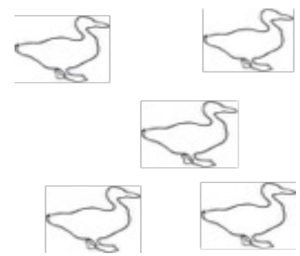
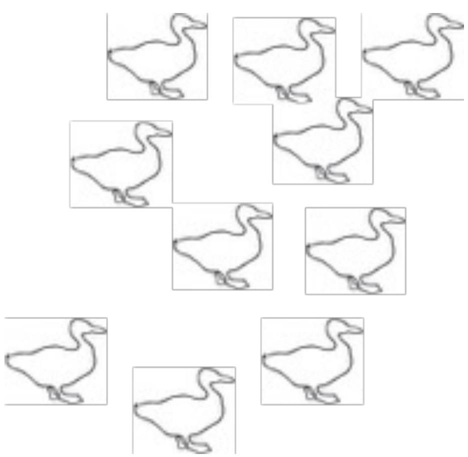


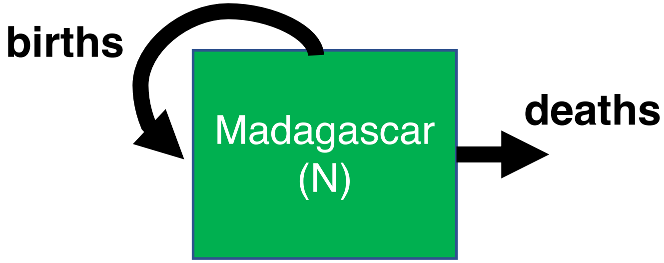
starting population

if stochastic? "up to chance"



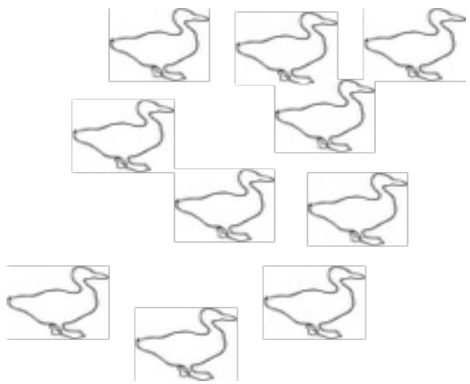
probability of death = 0.5

The basic population model



starting population

if deterministic "always the same"



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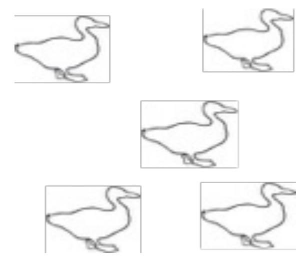
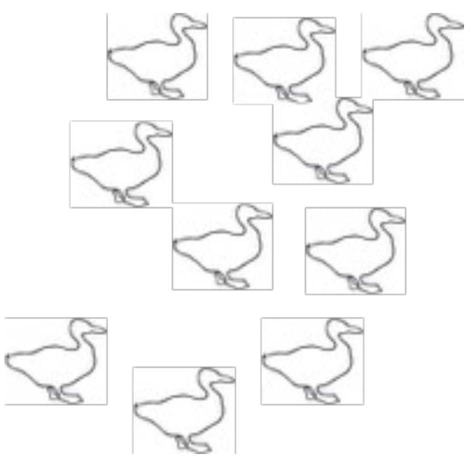


starting population

if stochastic? "up to chance"



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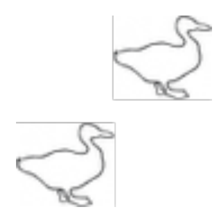

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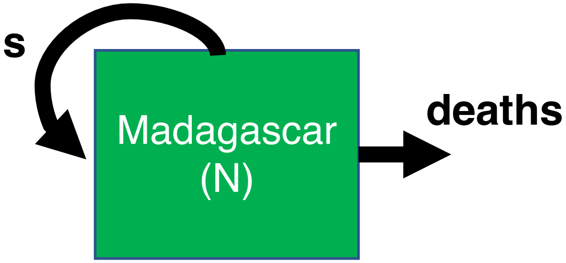
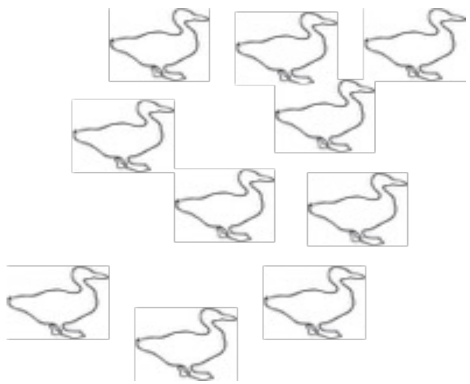


starting population



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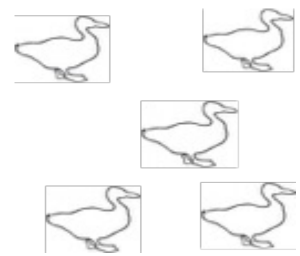

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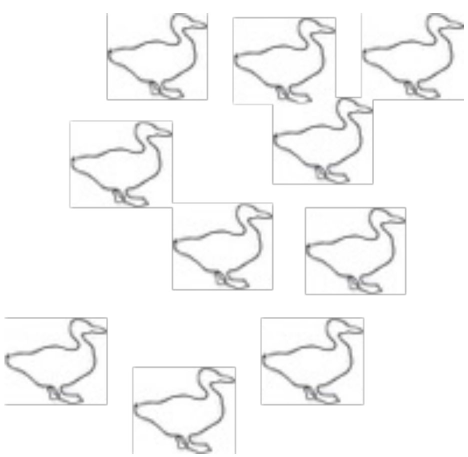

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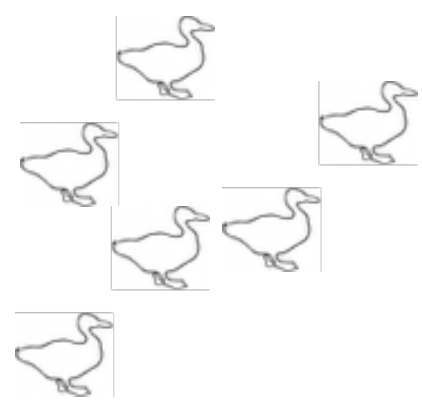

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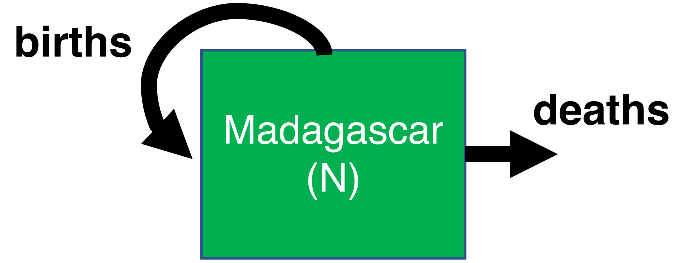
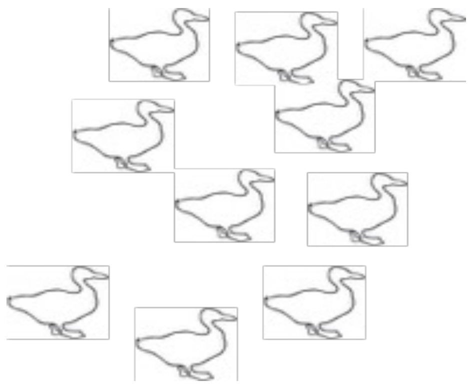


starting population



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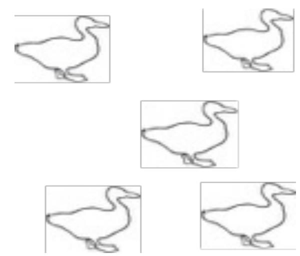

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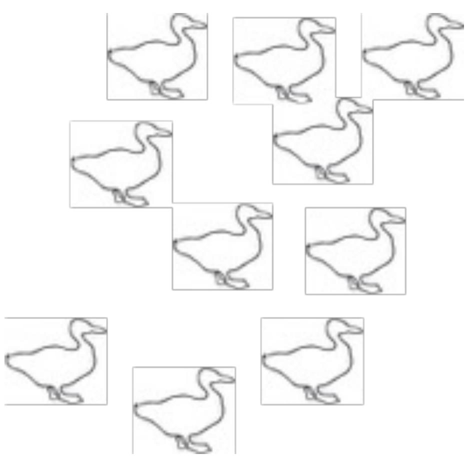
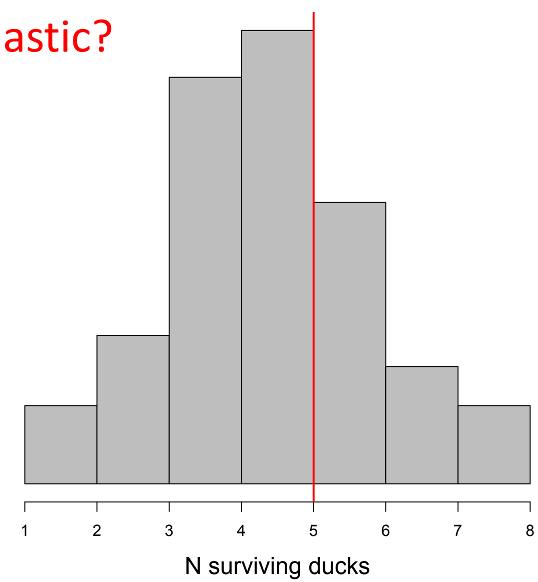


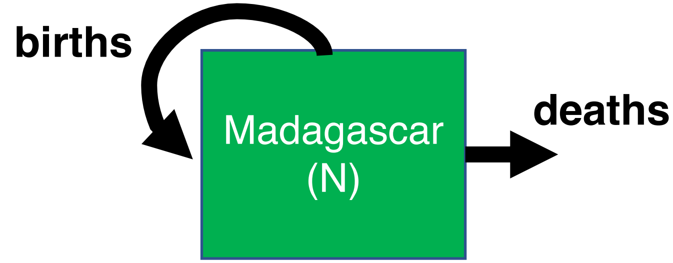
probability of death = 0.5

rbinom(200,10,0.5)

if stochastic?



The basic population model



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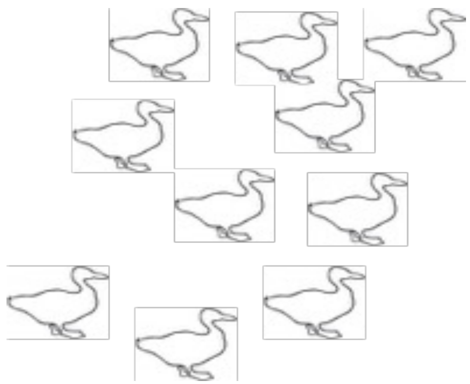

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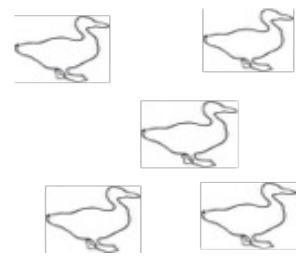
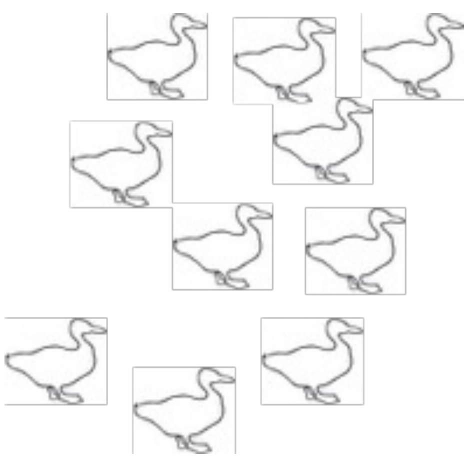


If you test your 10 ducks many times, on average you get 5

rbinom(200,10,0.5)

starting population



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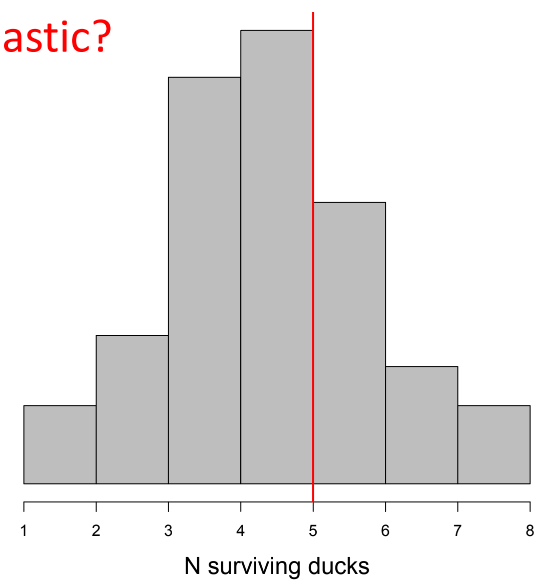


What is the difference between deterministic and stochastic?

Deterministic = always the same
Stochastic = up to chance

Key concepts

- Compartmental/mechanistic/mathematical models

  Modèles en compartiments

- Continuous vs. discrete models

  Modèles en temps continue vs. modèles en temps discrète

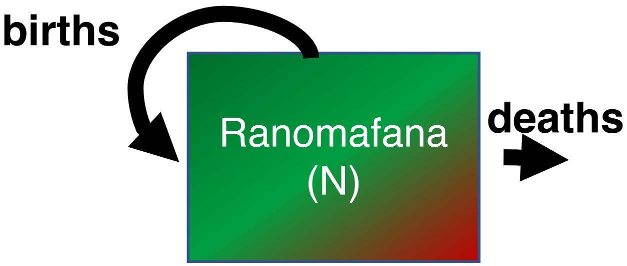
- Deterministic vs. stochastic models

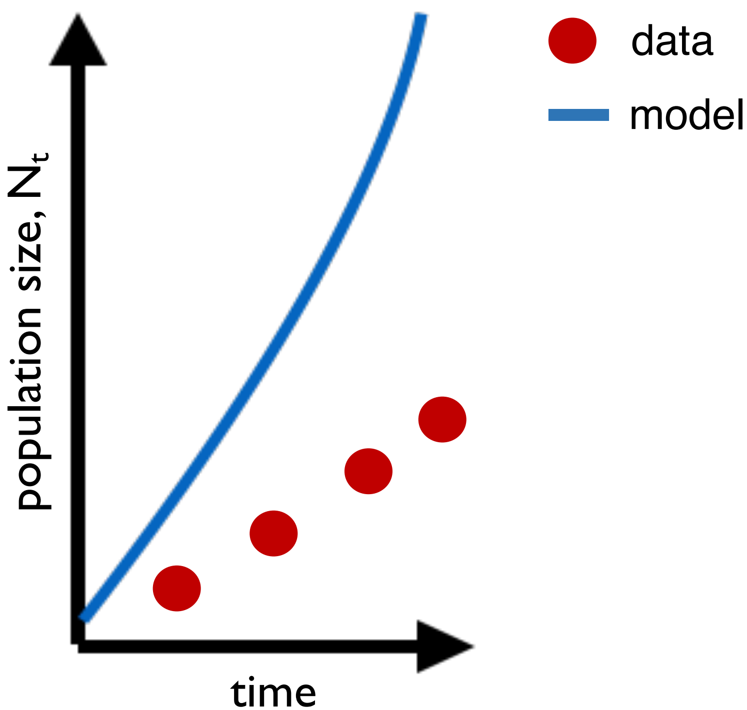
  Modèles détérministique vs. stochastique

2. Structured Population Models

2. Modèles de la population structurée

The structured population model

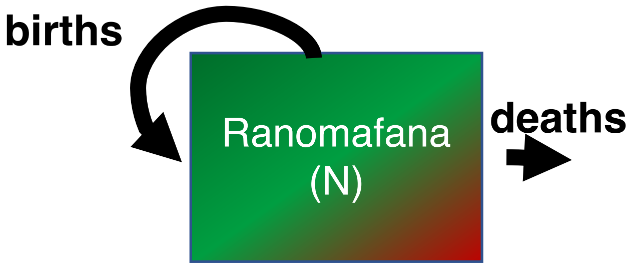


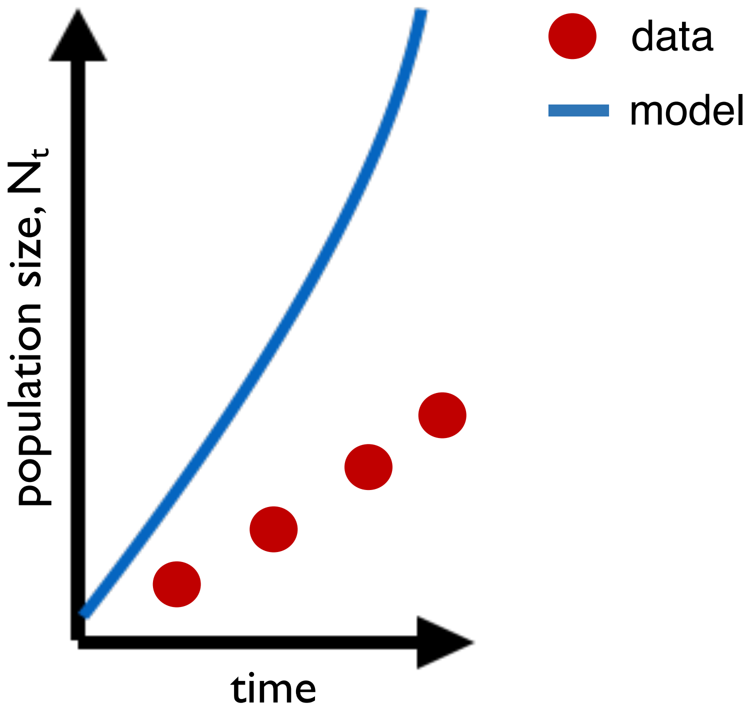


Why does the model perform poorly?

The basic population model

Why does the model perform poorly?





We need population structure!

That means distinguishing babies from adults.

The structured population model

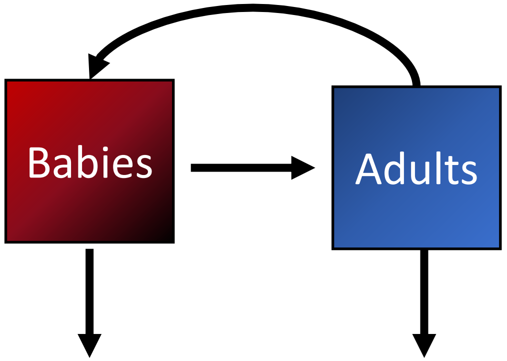
Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically

How does the population of Ranomafana grow over time?

Comment est-ce que la population de Ranomafana s'augmente avec le passage du temps?

The structured population model



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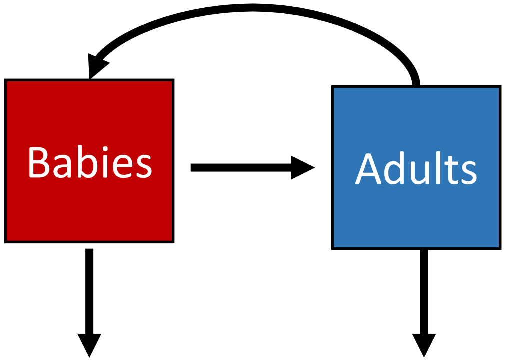

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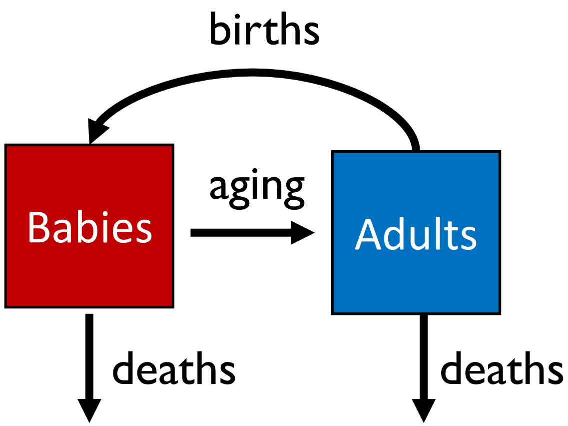

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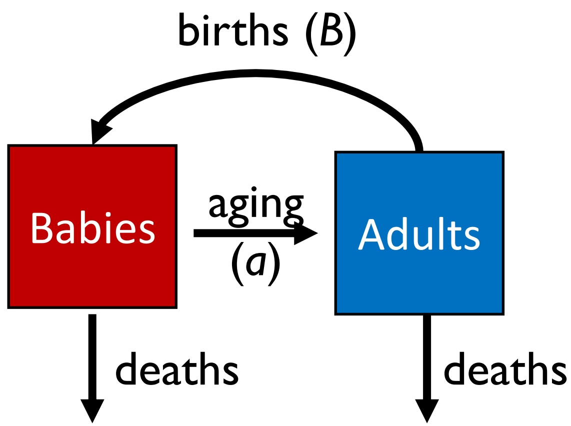

$$N_{t+1} = \lambda * N_t$$

pop size at t + 1

pop size at t

Population rate of increase

Taux d'accroissement de la population

Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
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$$\mathbf{n_{t+1}} = \mathbf{A}\,\mathbf{n_t}$$

matrix of rates

vector of population sizes

***Discrete time**

The structured population model

Compartmental models (Mechanistic Models)

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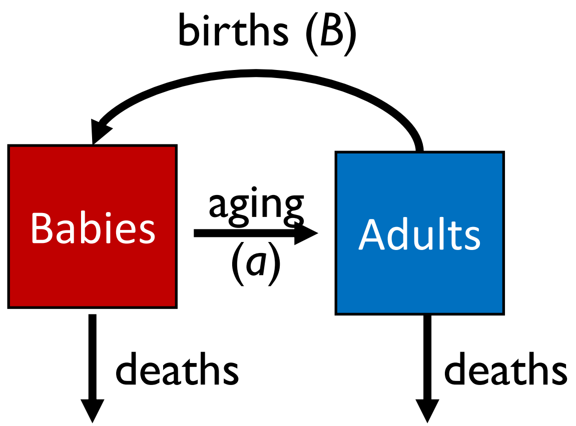

$$\mathbf{n_{t+1}} = \mathbf{A}\,\mathbf{n_t}$$

$$\underbrace{\begin{bmatrix} s_b(1-a) & B \\ s_b a & s_a \end{bmatrix}}_{\mathbf{A}} \times \underbrace{\begin{bmatrix} n_b \\ n_a \end{bmatrix}}_{\mathbf{n_t}} = \underbrace{\begin{bmatrix} s_b(1-a)\, n_b + b n_a \\ s_b\, a\, n_b + s_a n_a \end{bmatrix}}_{\mathbf{n_{t+1}}}$$

The structured population model

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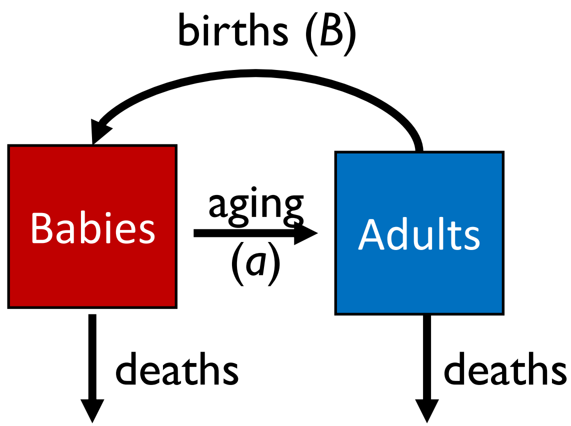

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Population growth will depend on population structure!

Key concepts

- Compartmental/mechanistic/mathematical models

  Modèles en compartiments

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- Structured models

  Modèles structurés.

How do we modify a basic population model to make it structured?

- Two compartments (adults and babies)
- Vector/matrix of values

3. Two-population model

3. modèles de deux populations

The predator-prey model

Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
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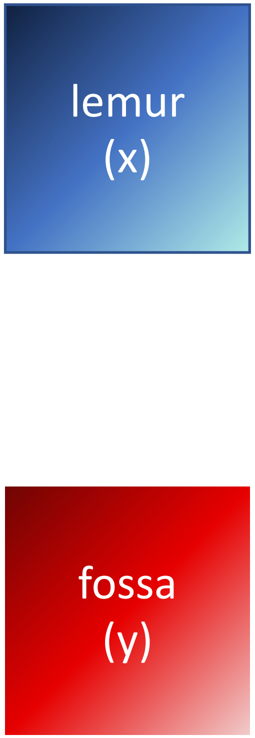
4. Rates of transferring between compartments are expressed mathematically

How does the population of fossa regulate the population of lemurs in Ranomafana?

Comment la population de "fossa" régule la population de lemuriens à Ranomafana?

The predator-prey model



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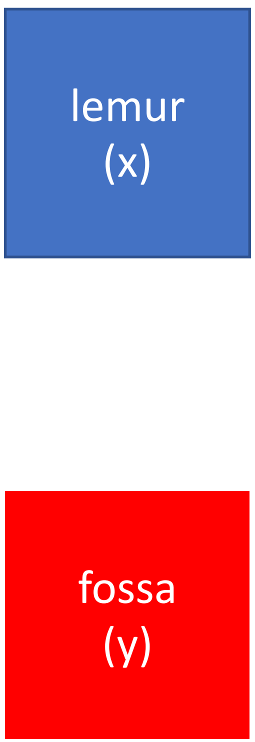

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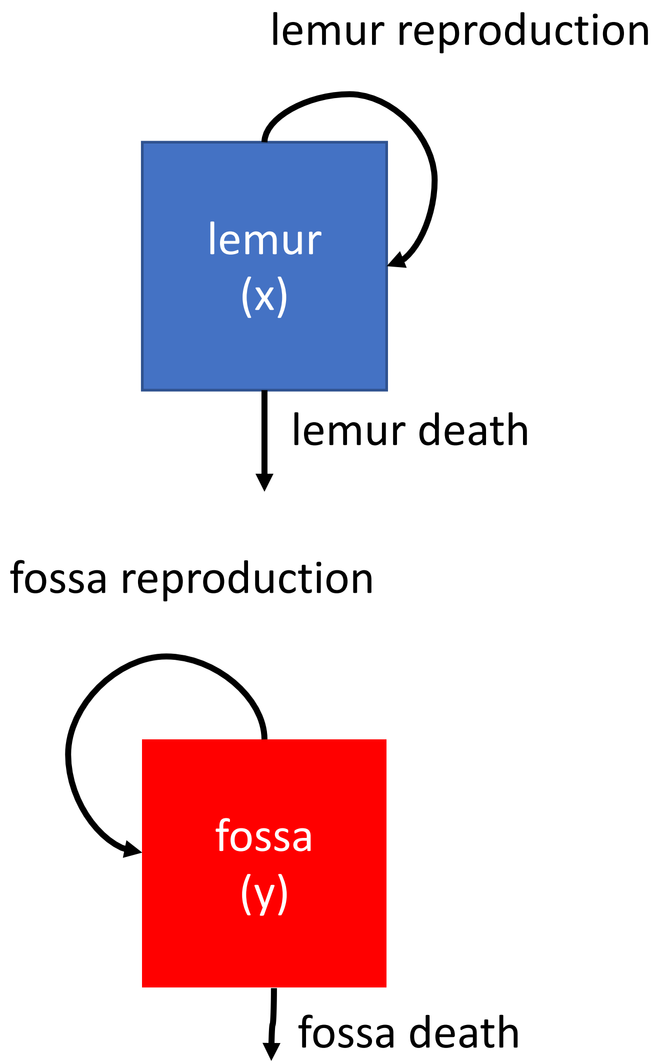

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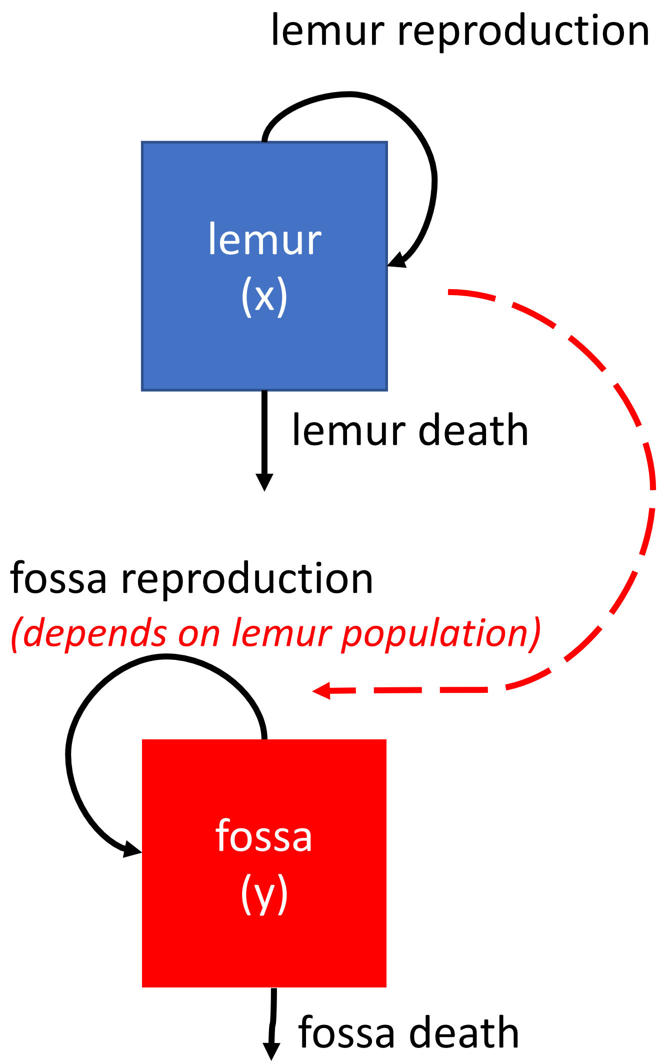

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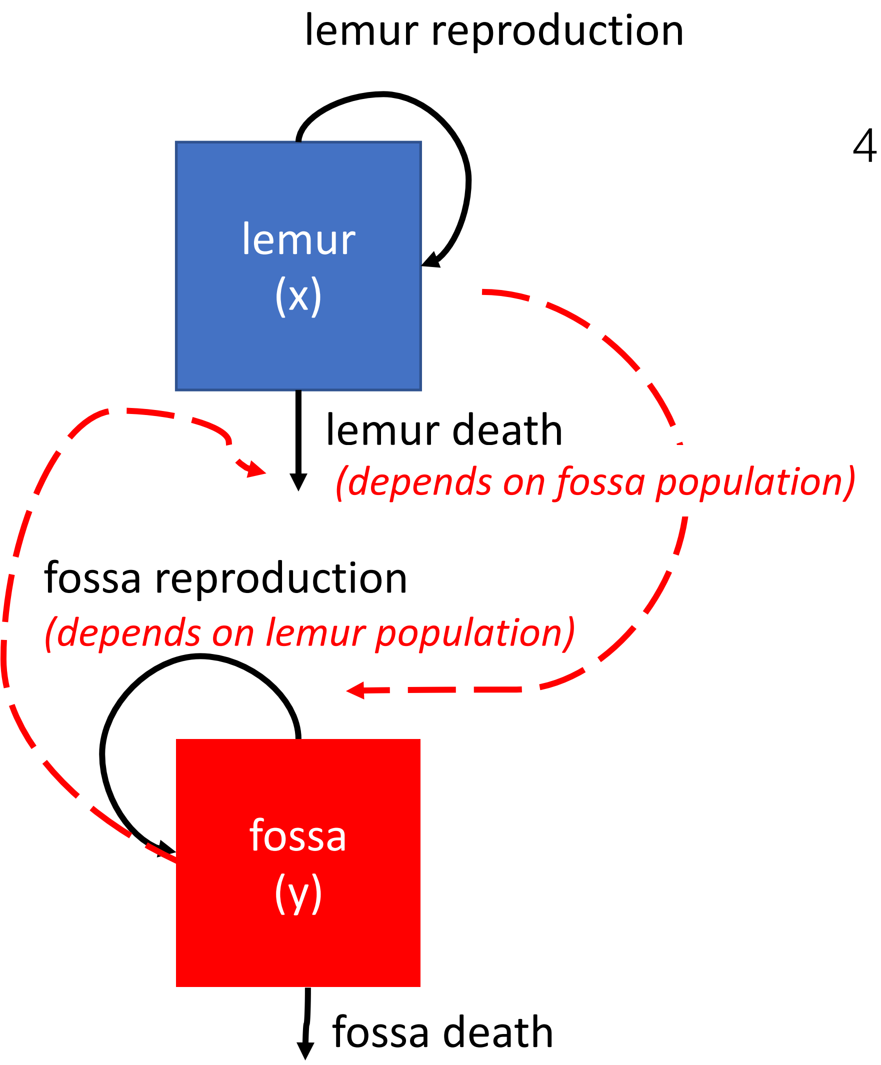

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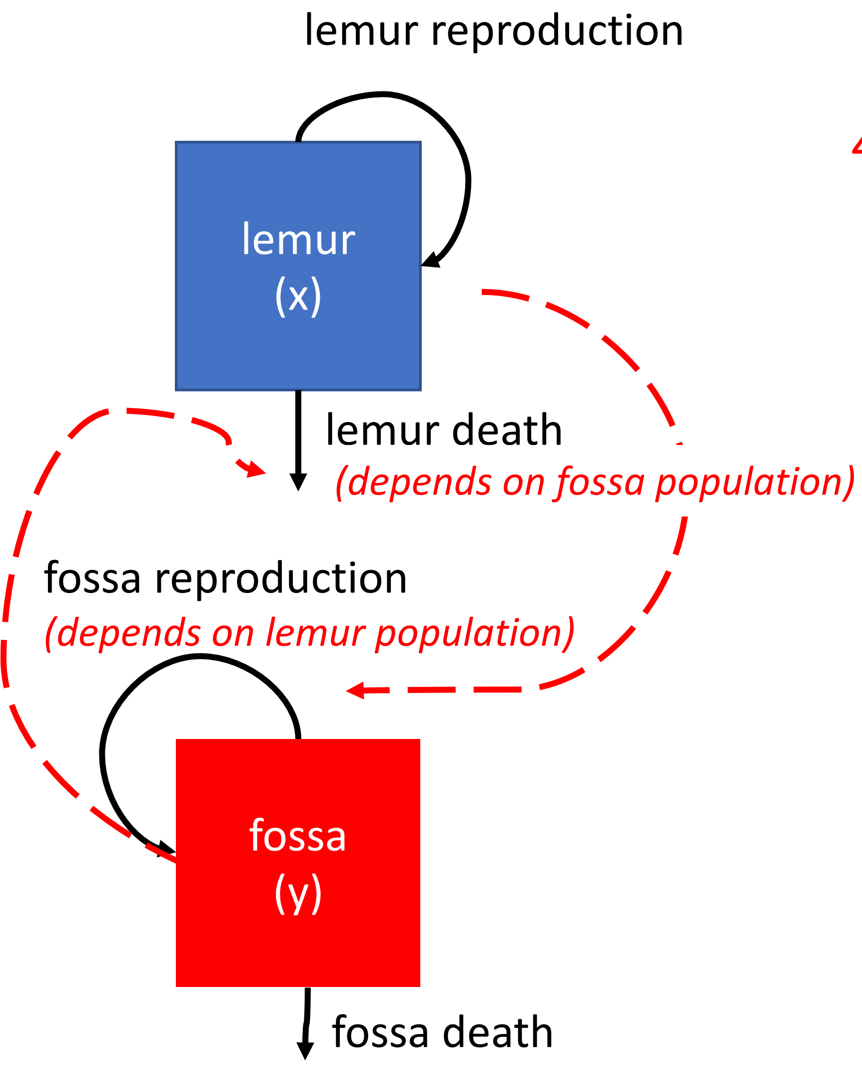

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Parameters

: lemur rep. rate

: lemur death rate

: fossa rep. rate

: fossa death rate

The predator-prey model

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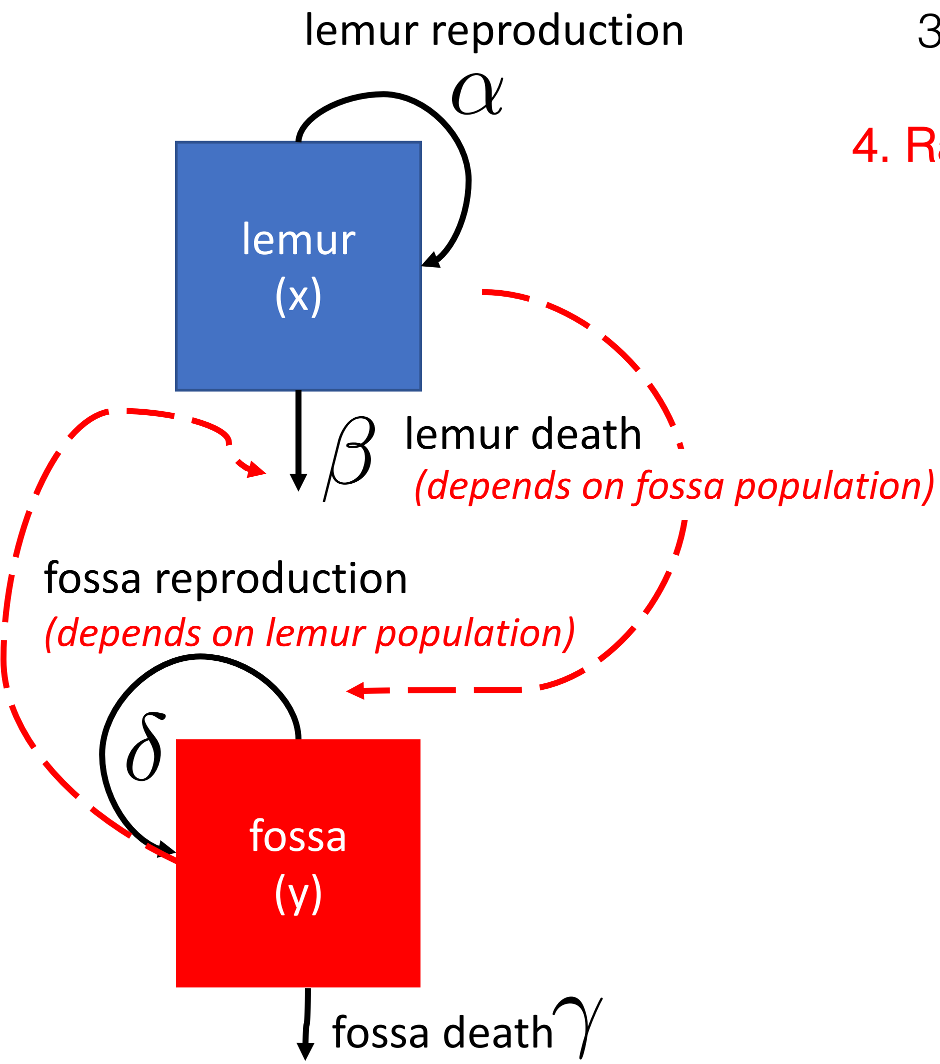


Parameters

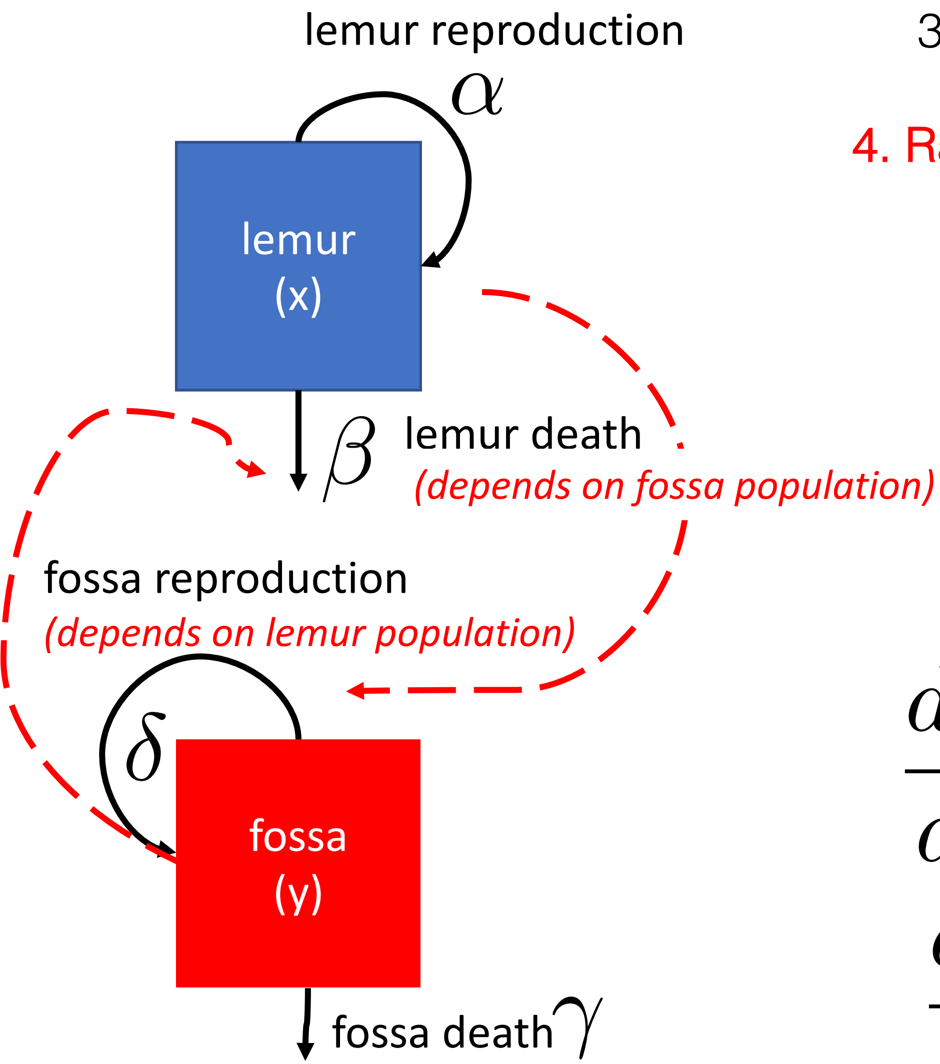
α : lemur rep. rate

β : lemur death rate

δ : fossa rep. rate

γ : fossa death rate

The predator-prey model



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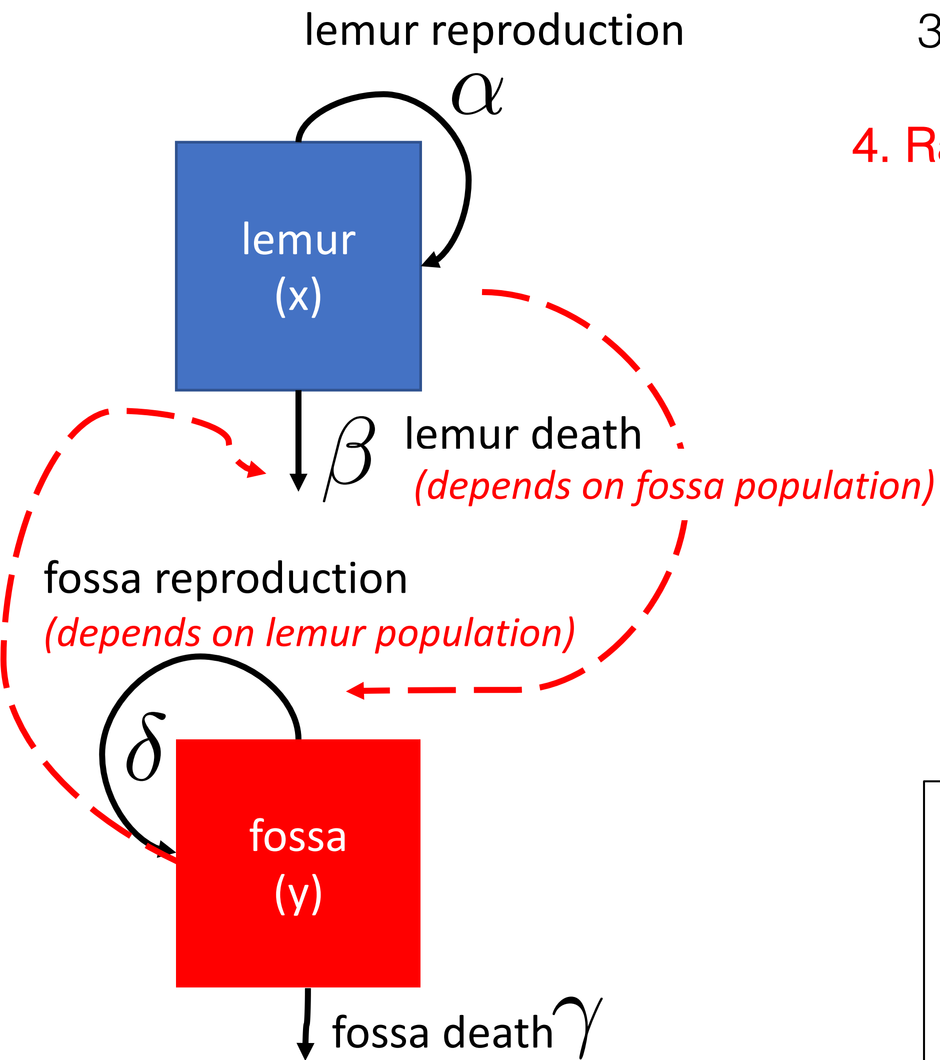
$$\frac{dx}{dt} = x(\alpha - \beta y)$$

$$\frac{dy}{dt} = y(\delta x - \gamma)$$

The predator-prey model



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$$\frac{dx}{dt} = x(\alpha - \beta y)$$

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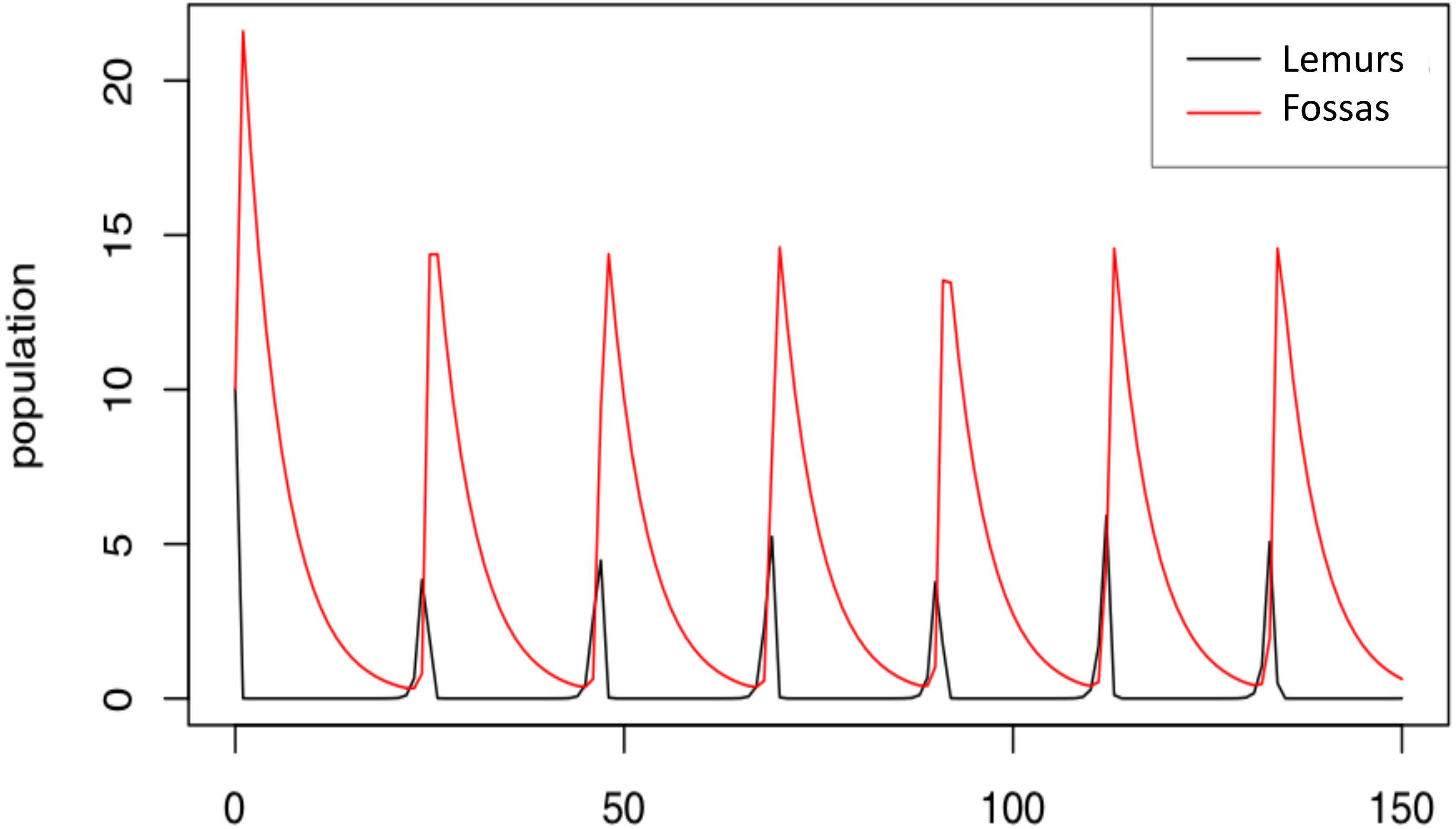
SOME ASSUMPTIONS

- the **lemur** has an unlimited food supply
- the **lemur** only dies from being eaten by fossa
- the **fossa** is totally dependent on a single prey species (the lemur) as its only food supply

The predator-prey model



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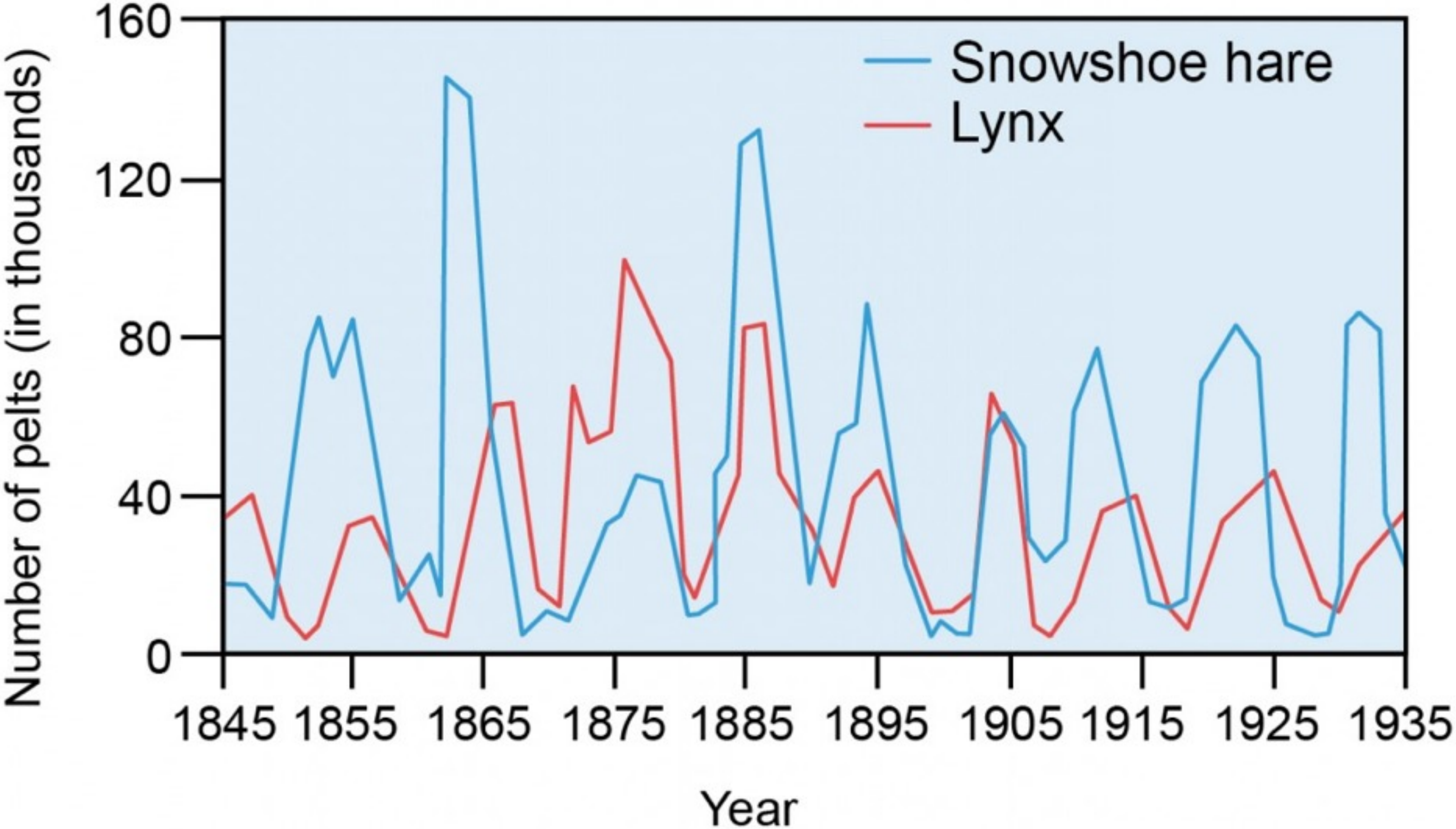

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- Structured models

  Modèles structurés.

- Two population models

  Modèles des deux populations

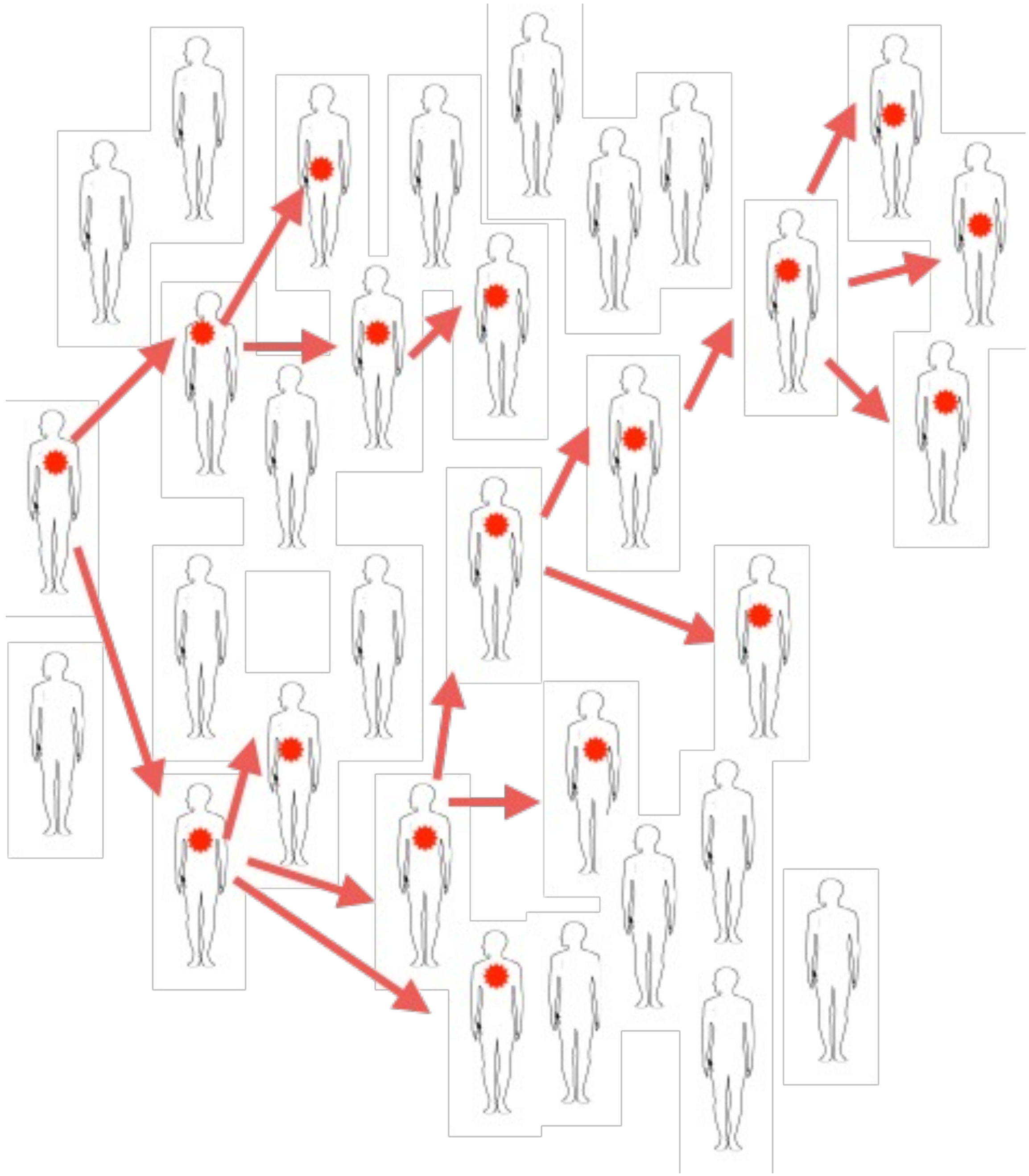
What pattern can we see in simple predator-prey relationships?

Cycles / oscillations

What could we modify to make this model more complex/realistic?

Lemurs/fossa eat other things, die of other causes

4. SIR Models
4. Les modèles SIR



The SIR model

Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
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How does measles transmit through Antananarivo?

Comment la rougéole se transmet-elle à Antananarivo?

The SIR model

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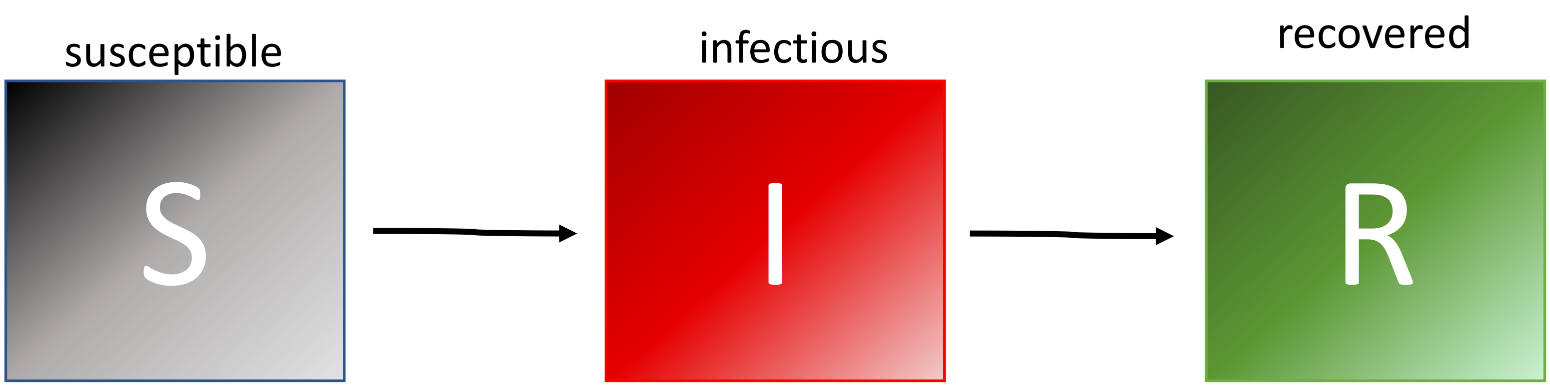

The SIR model

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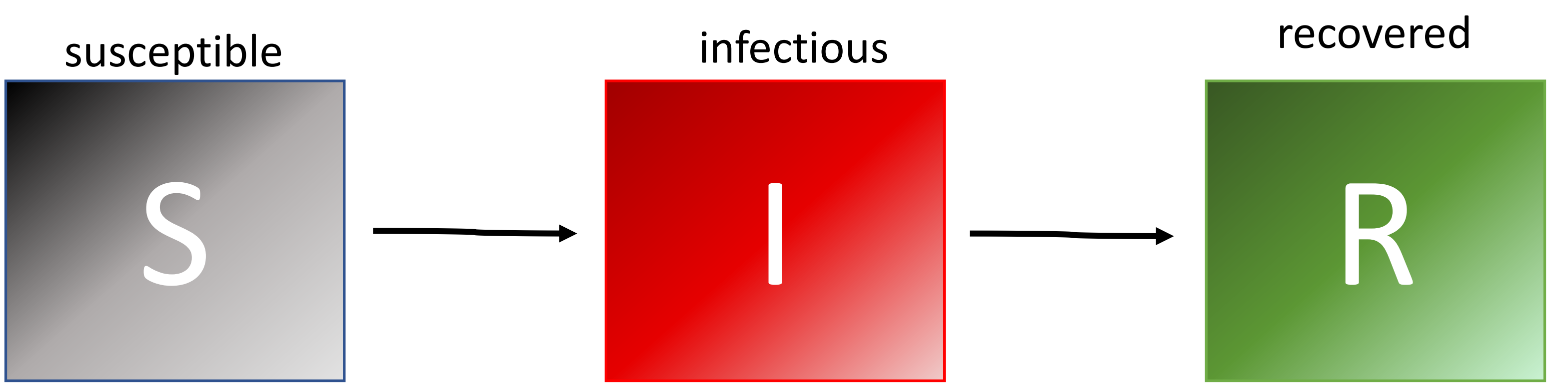

What are the big assumptions here?

The SIR model

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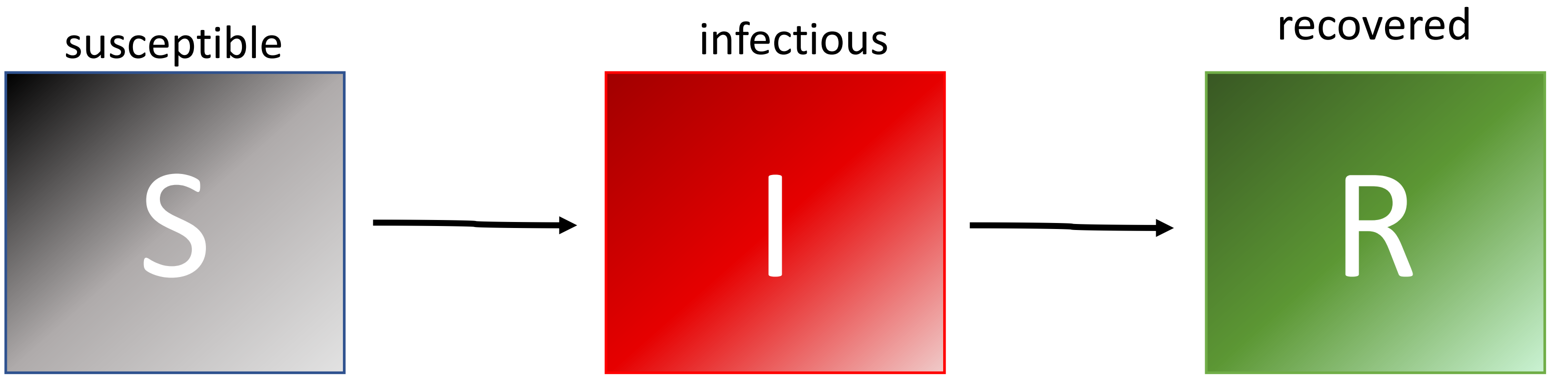
everyone is either:



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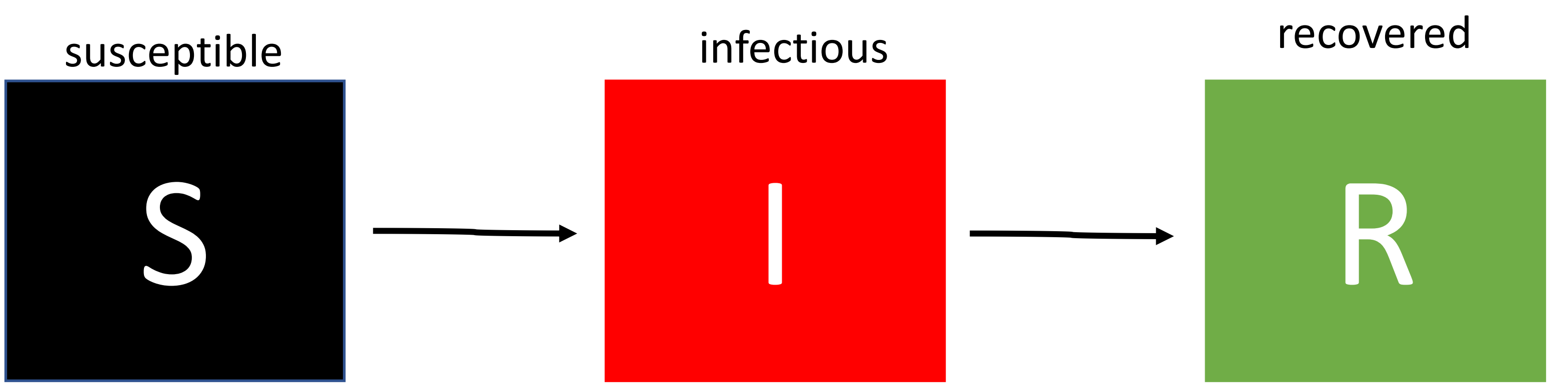

people mix uniformly (mass action)

les gens se mélangent uniformément

The SIR model

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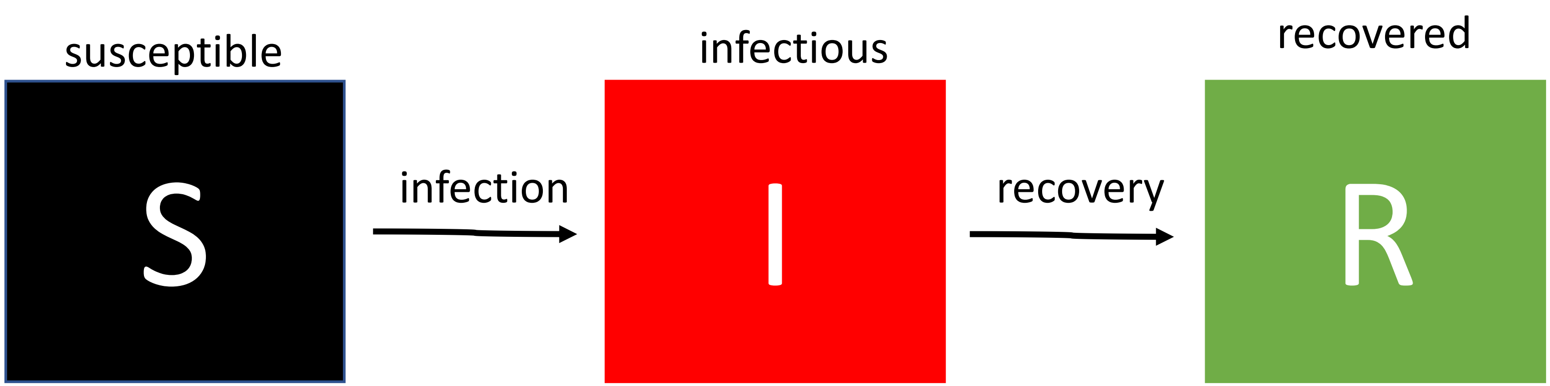

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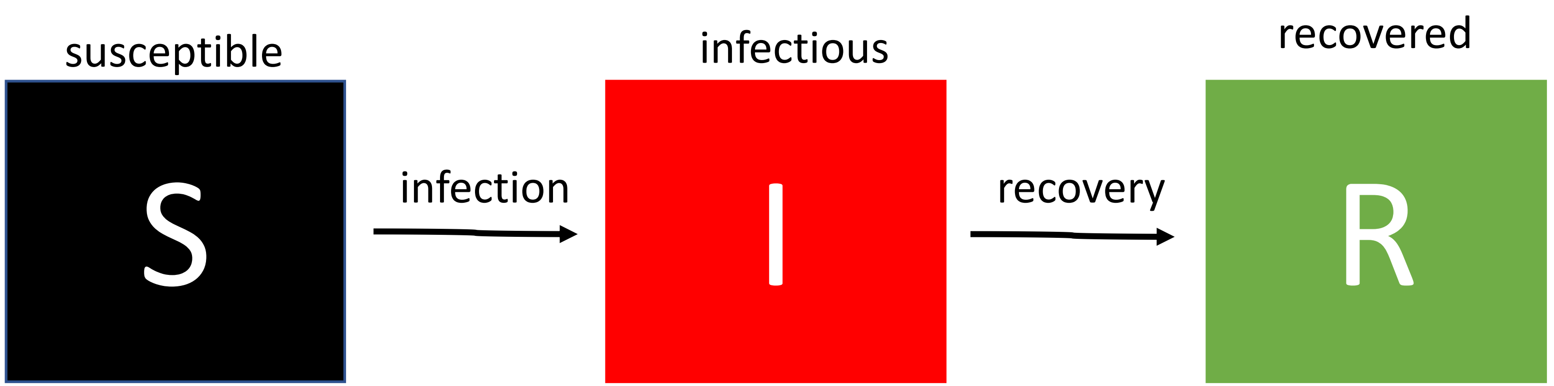
(mass action)
les gens se mélangent uniformément

no latent period
(infectious when infected)
pas de période de latence

The SIR model

Compartmental models (Mechanistic Models)

1. Populations are divided into compartments
2. Individuals within a compartment are homogenously mixed
3. Compartments and transition rates are determined by biological systems
4. Rates of transferring between compartments are expressed mathematically



recovery is permanent

la récupération est permanente

people mix uniformly (mass action)

les gens se mélangent uniformément

no latent period (infectious when infected)

pas de période de latence

The SIR model

Compartmental models (Mechanistic Models)

population size constant - no births or deaths, migration

taille de population constante

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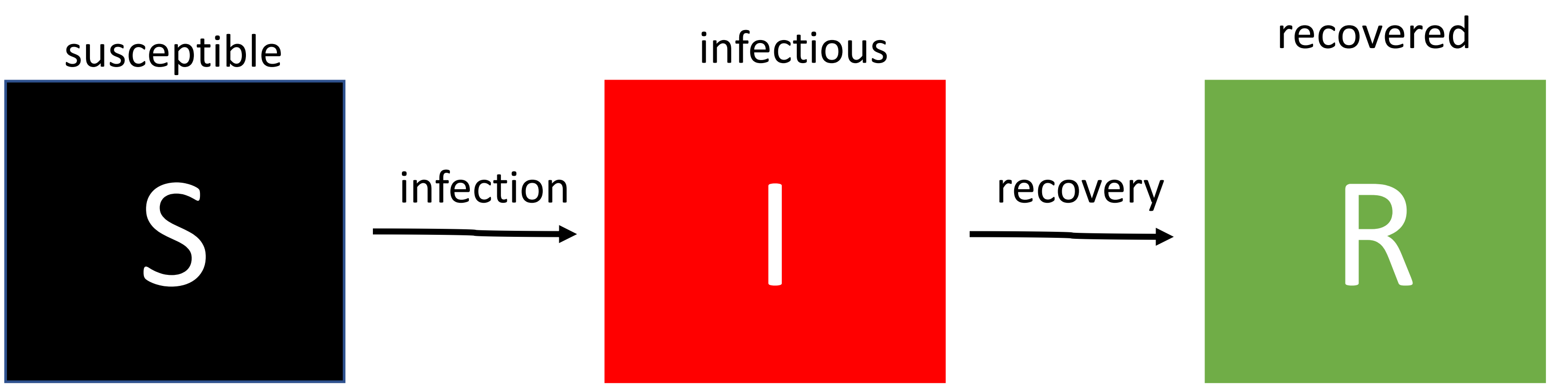

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The SIR model

Parameters
β: transmission rate
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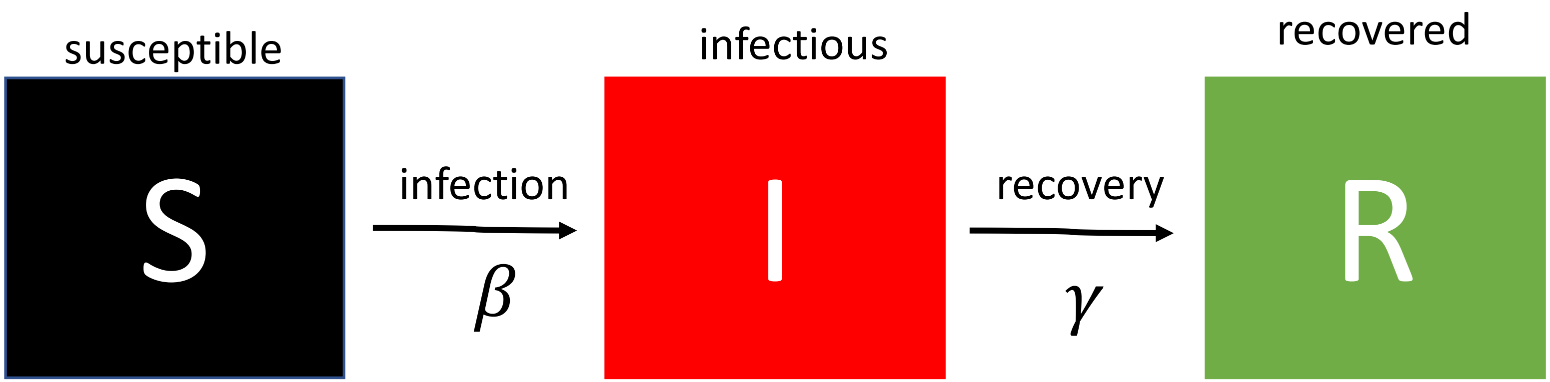

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t)$$

The SIR model

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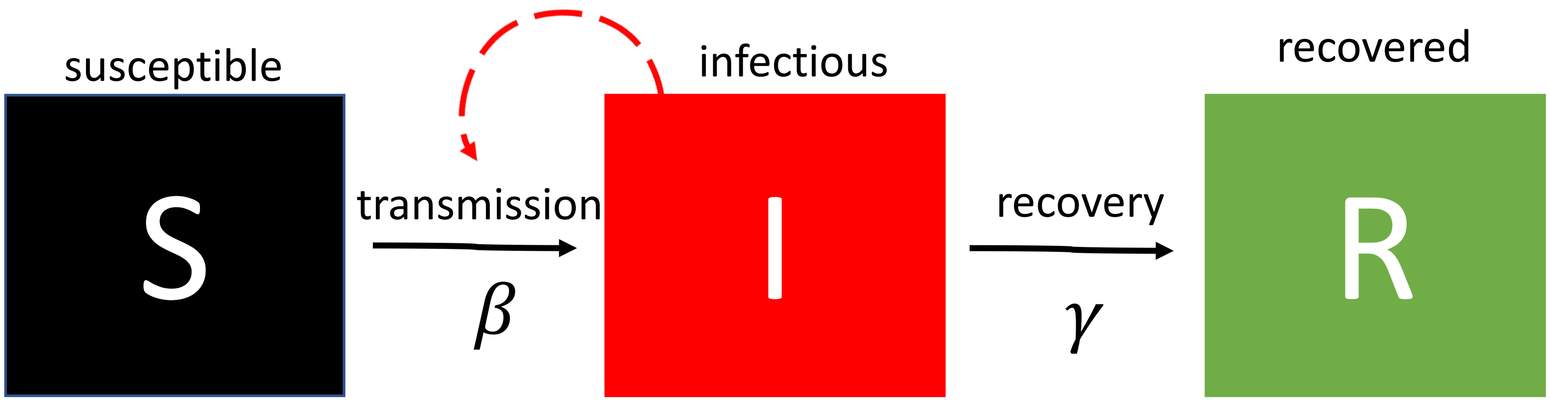

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…infected numbers influence the transmission rate….

The SIR model

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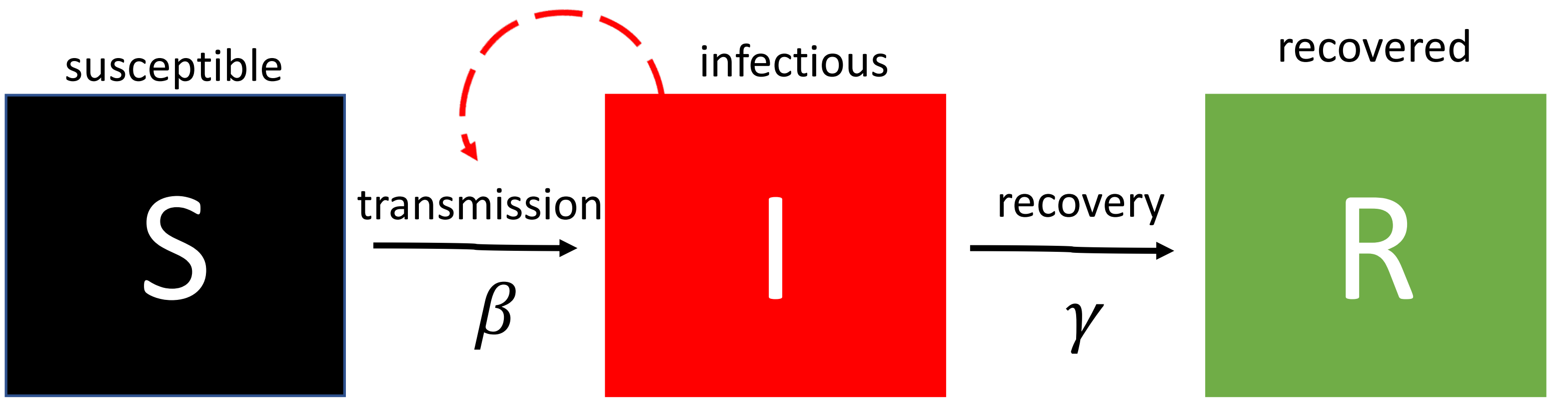
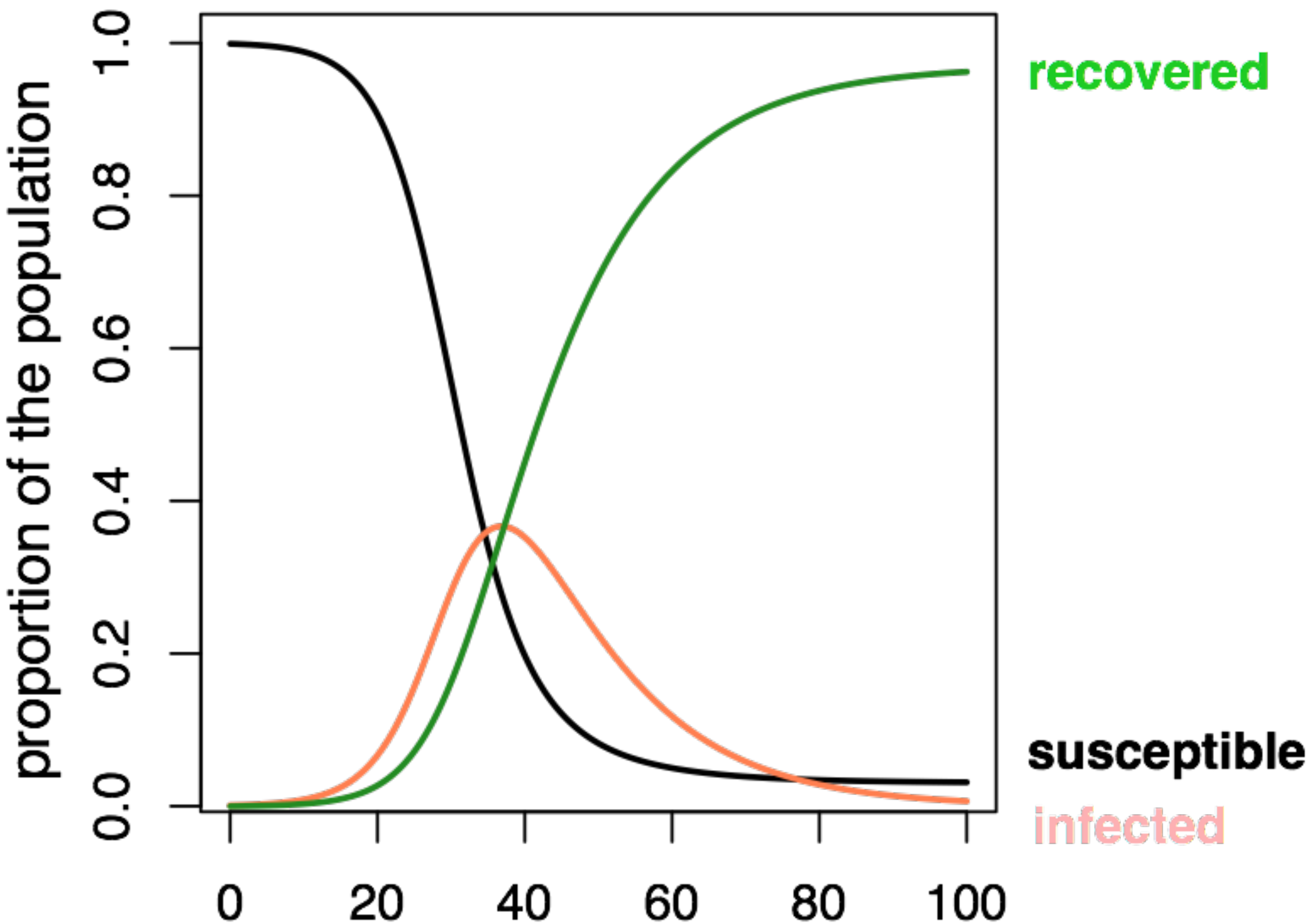
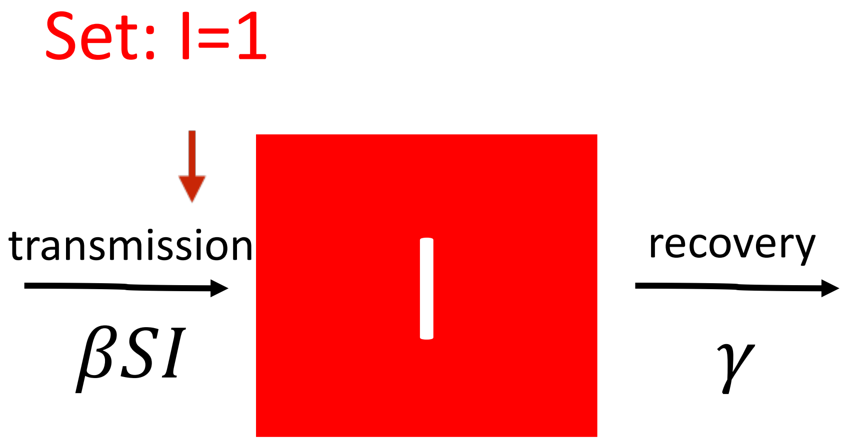


What will the dynamics look like?

The SIR model



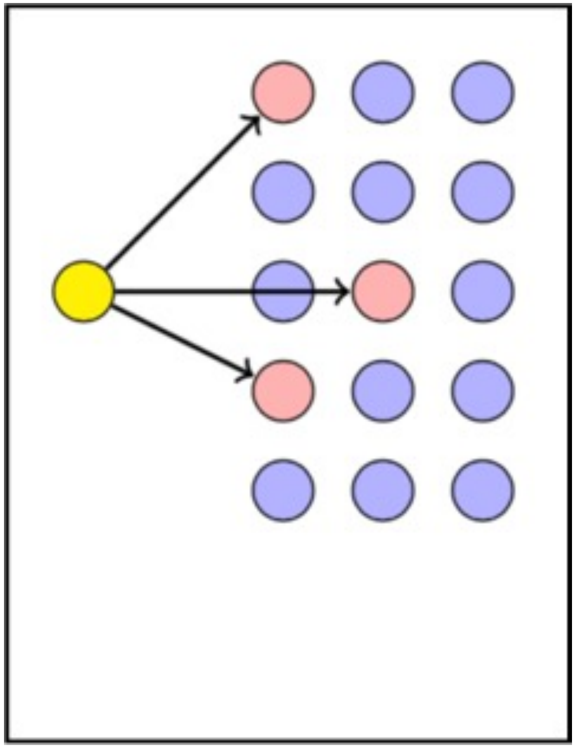
The SIR model



$$R_0 = \beta N / \gamma$$

The average number of persons infected by an infectious individual when everyone is susceptible (S=100%, or S=1, start of an epidemic)

The SIR model



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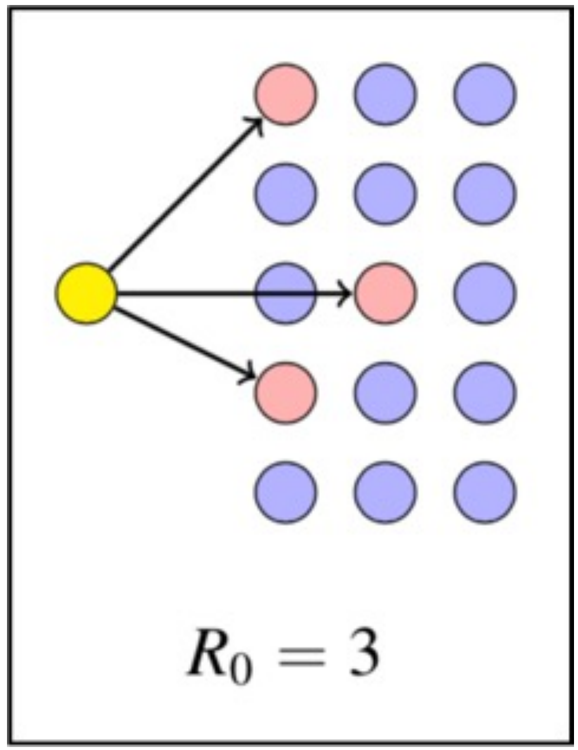

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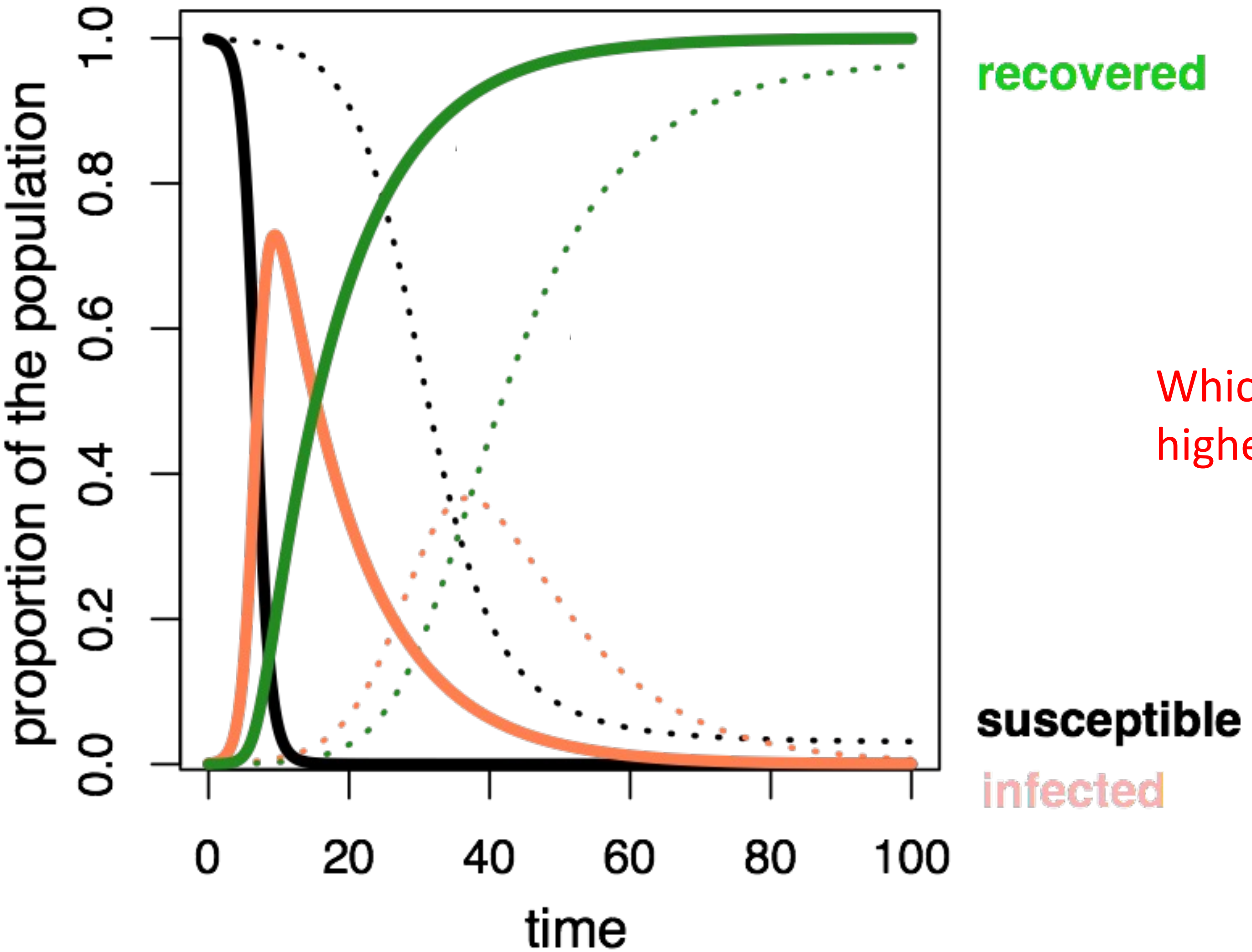
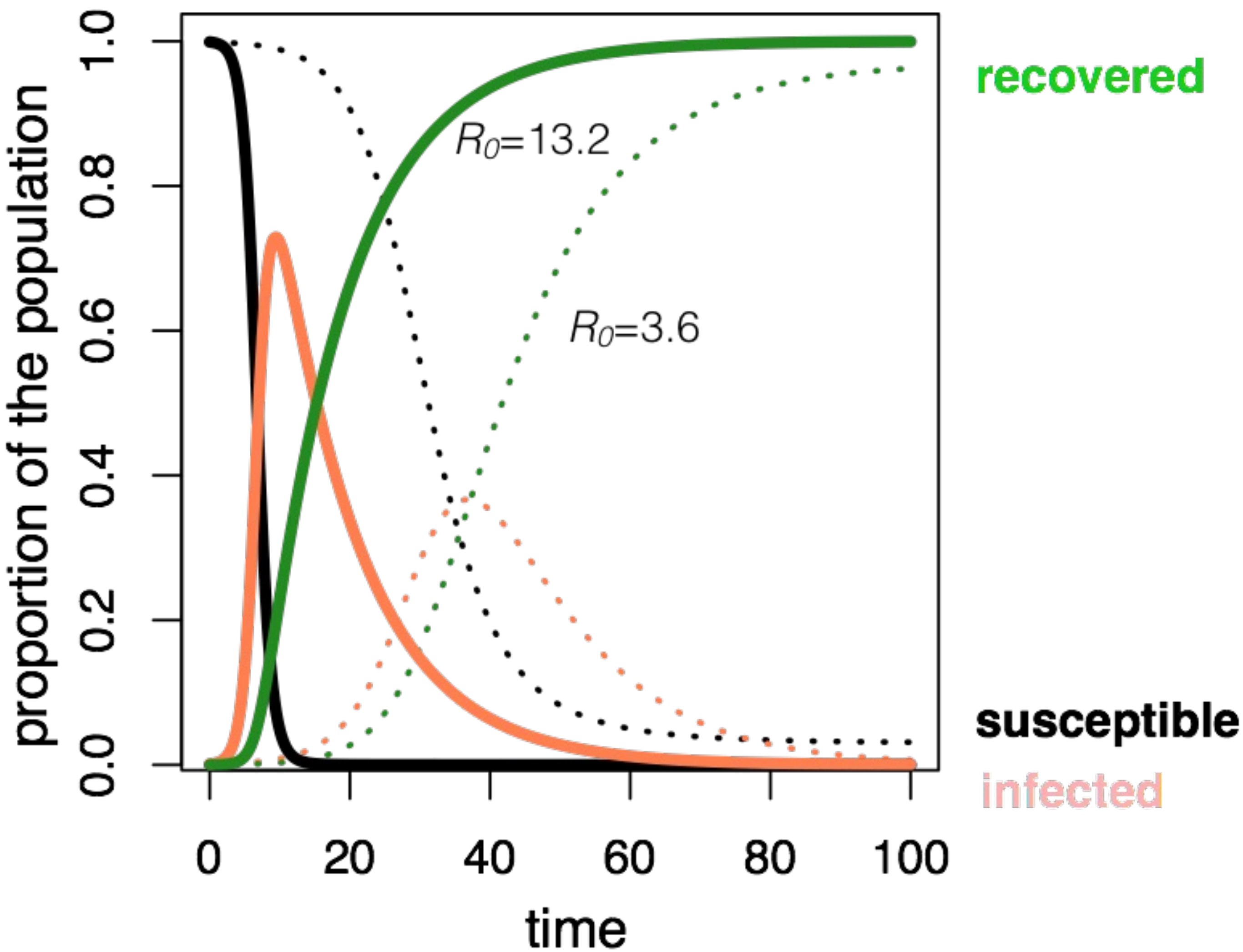


Which has the higher R0 value?

The SIR model

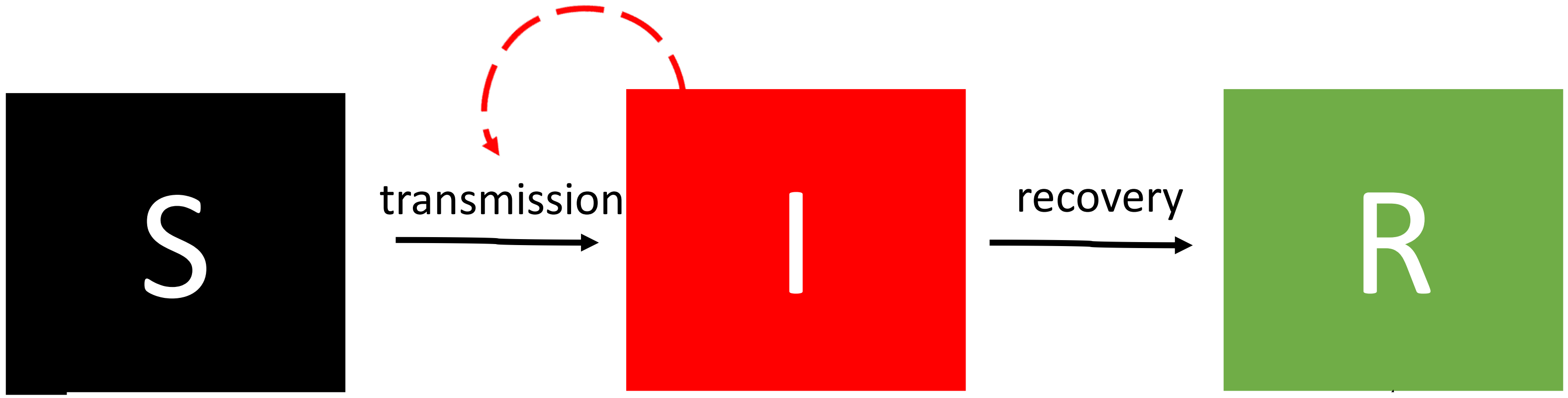


What is R0?

The average number of secondary infections from the first infectious individual

How could you modify this simple SIR model to represent COVID-19?

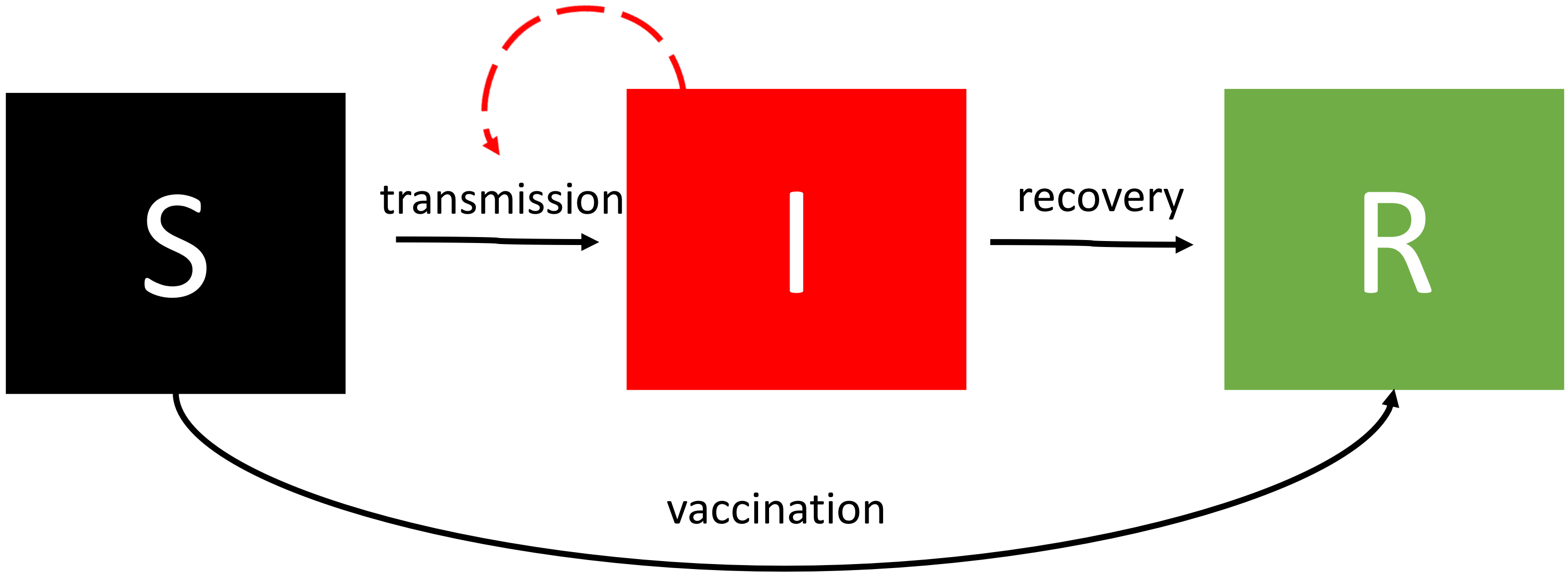
The SIR model : vaccination



Vaccination moves people out of susceptibles into the immune (recovered) class.

La vaccination éloigne les personnes sensibles de la maladie dans la classe immunitaire (rétablie).

The SIR model : vaccination



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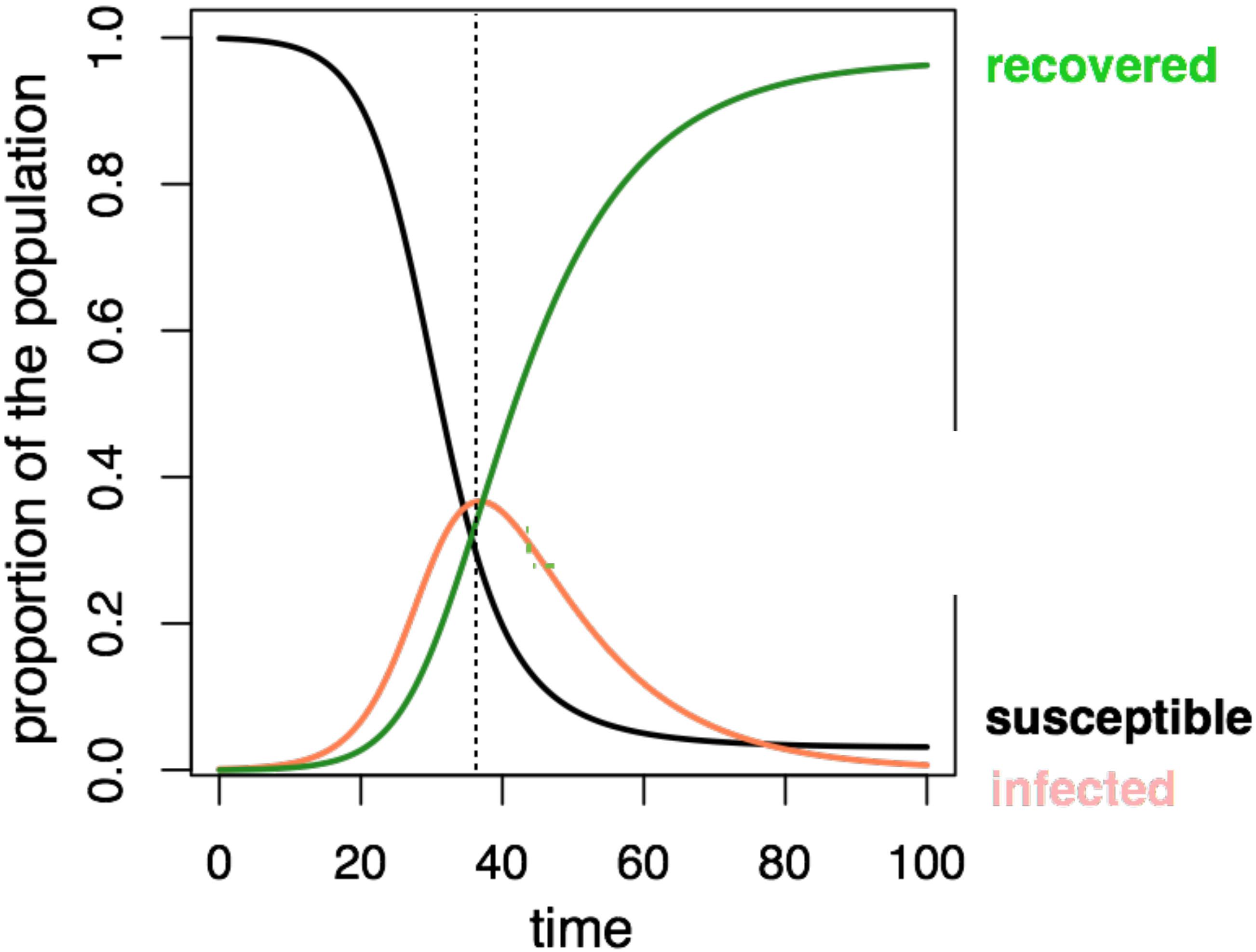
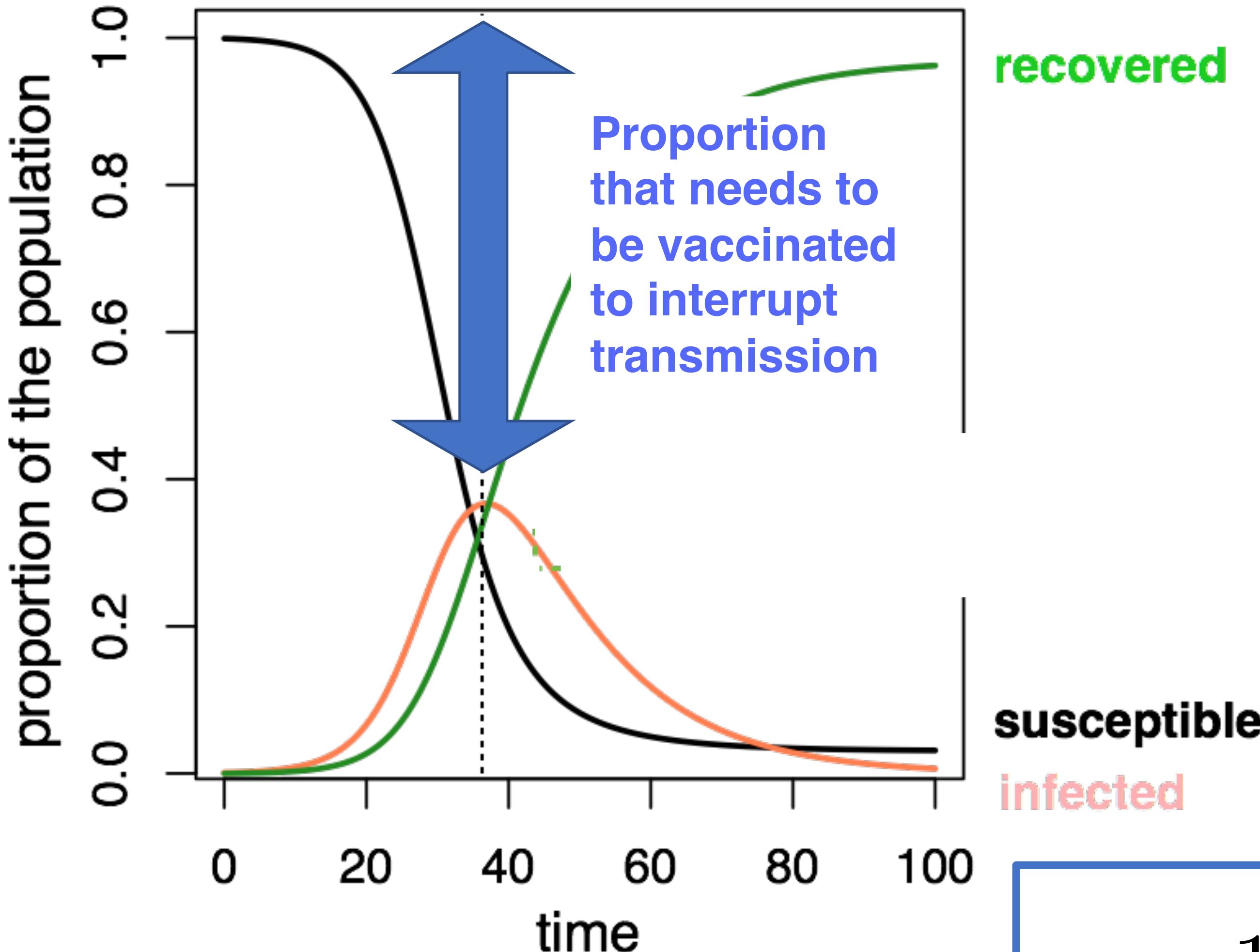


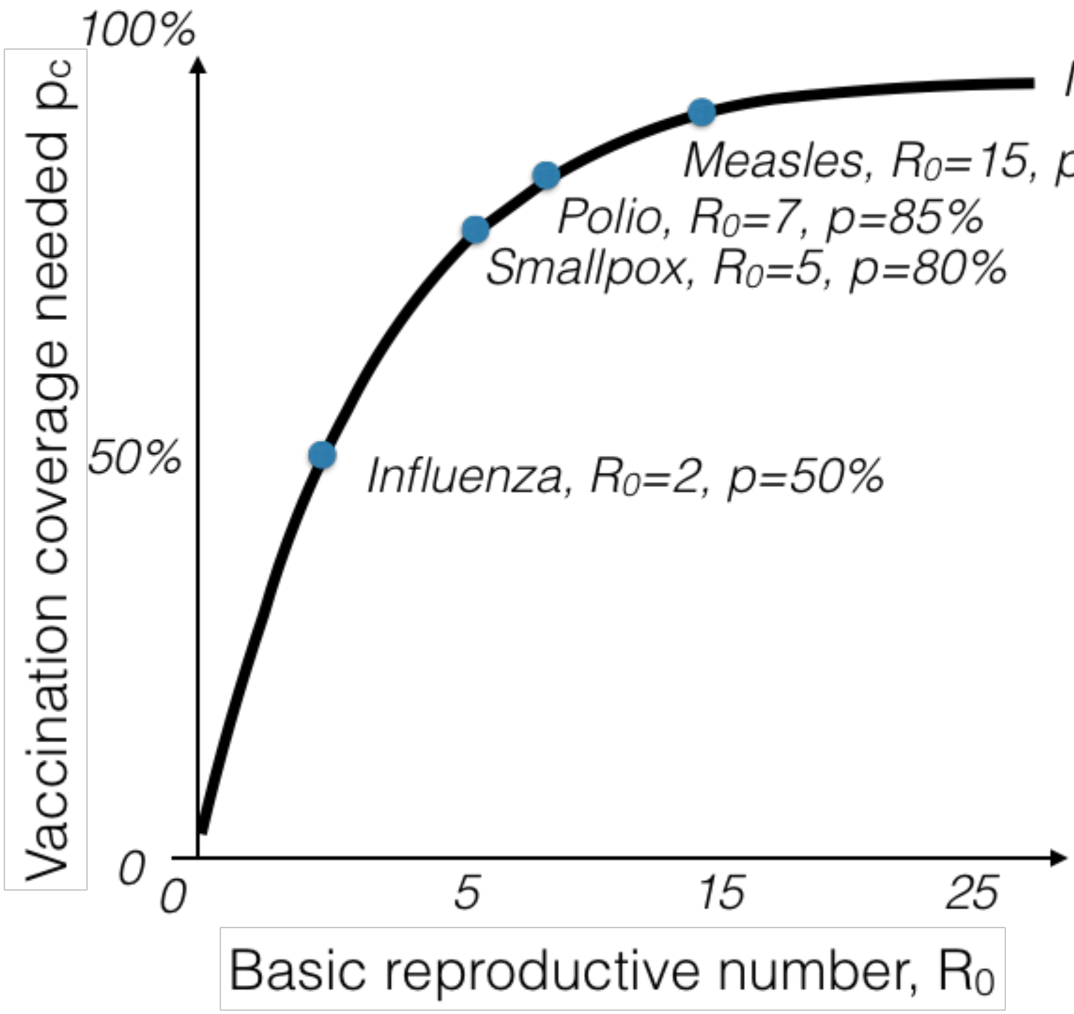
The SIR model : vaccination



$$p_c = 1 - \frac{1}{R_0}$$

The SIR model : eradication

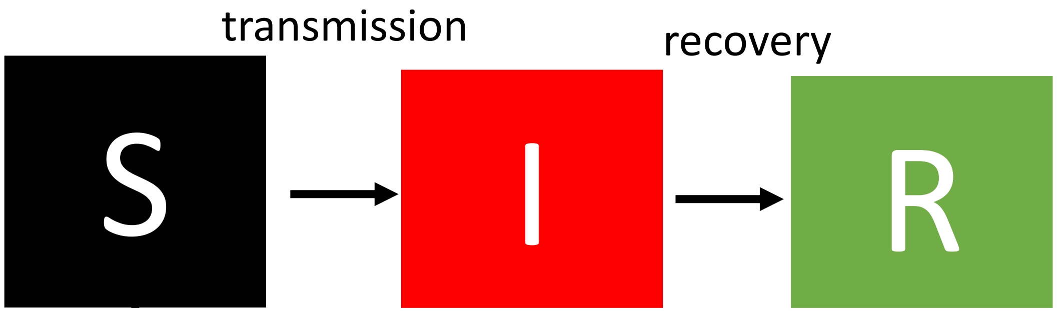
$$p_c = 1 - \frac{1}{R_0}$$



More transmissible diseases are harder to eradicate

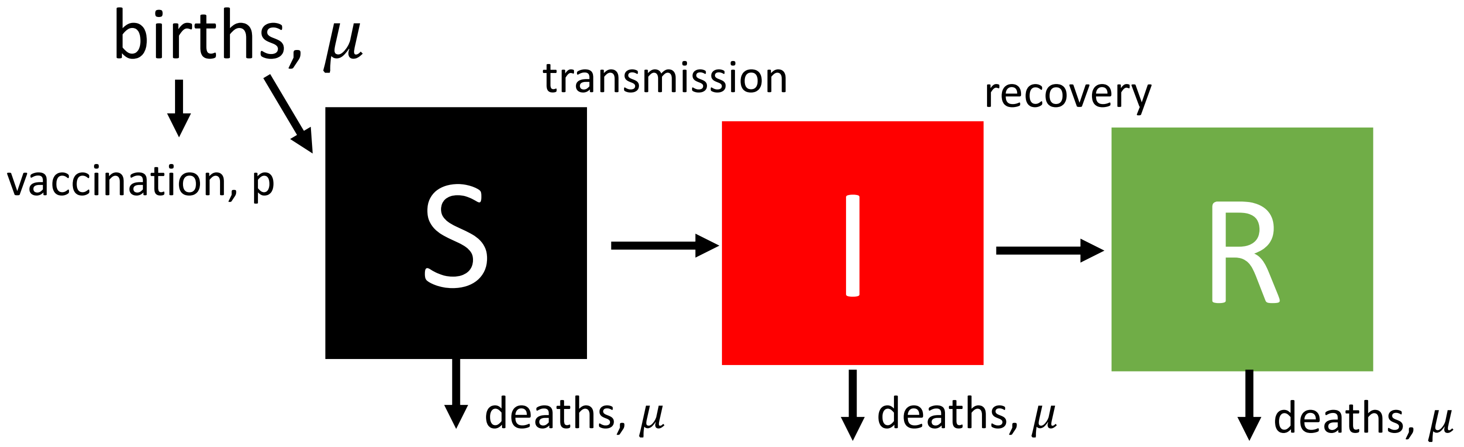
The SIR model : extensions

Moving beyond a ‘closed’ population



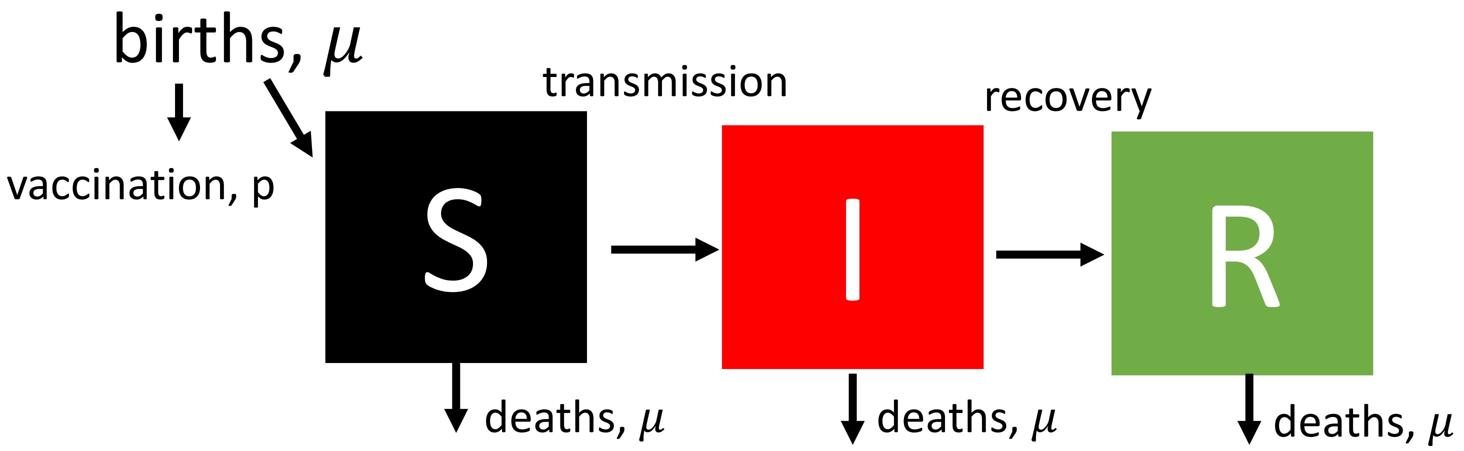
The SIR model : extensions

Moving beyond a 'closed' population



The SIR model : extensions

Moving beyond a ‘closed’ population

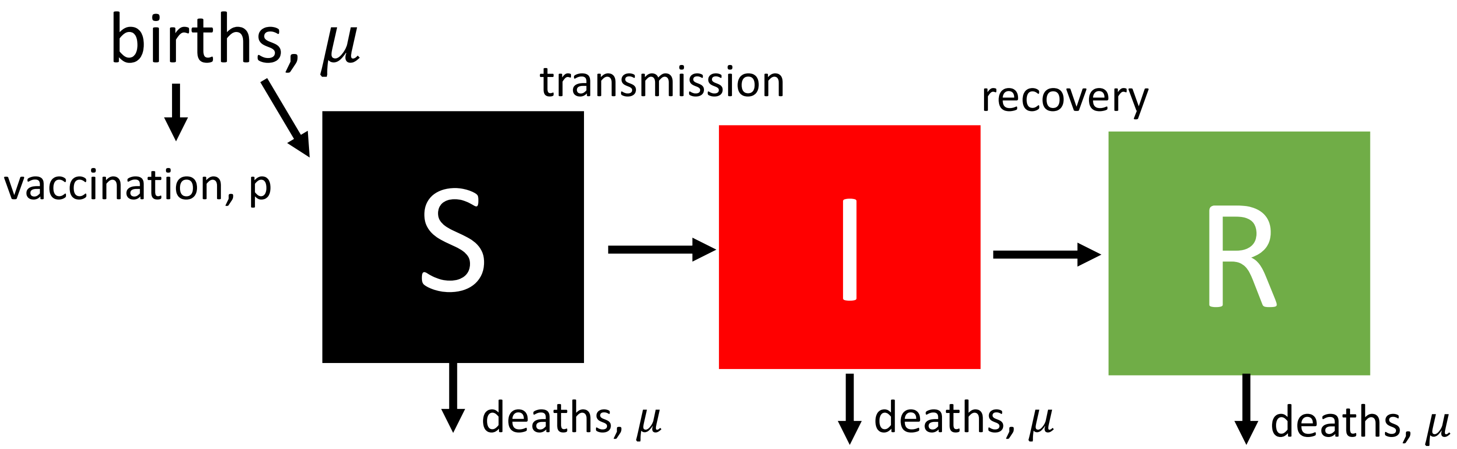


$$\frac{dS(t)}{dt} = \mu(1-p) - \beta S(t)I(t) - \mu S(t)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu I$$

The SIR model : add births

Moving beyond a 'closed' population



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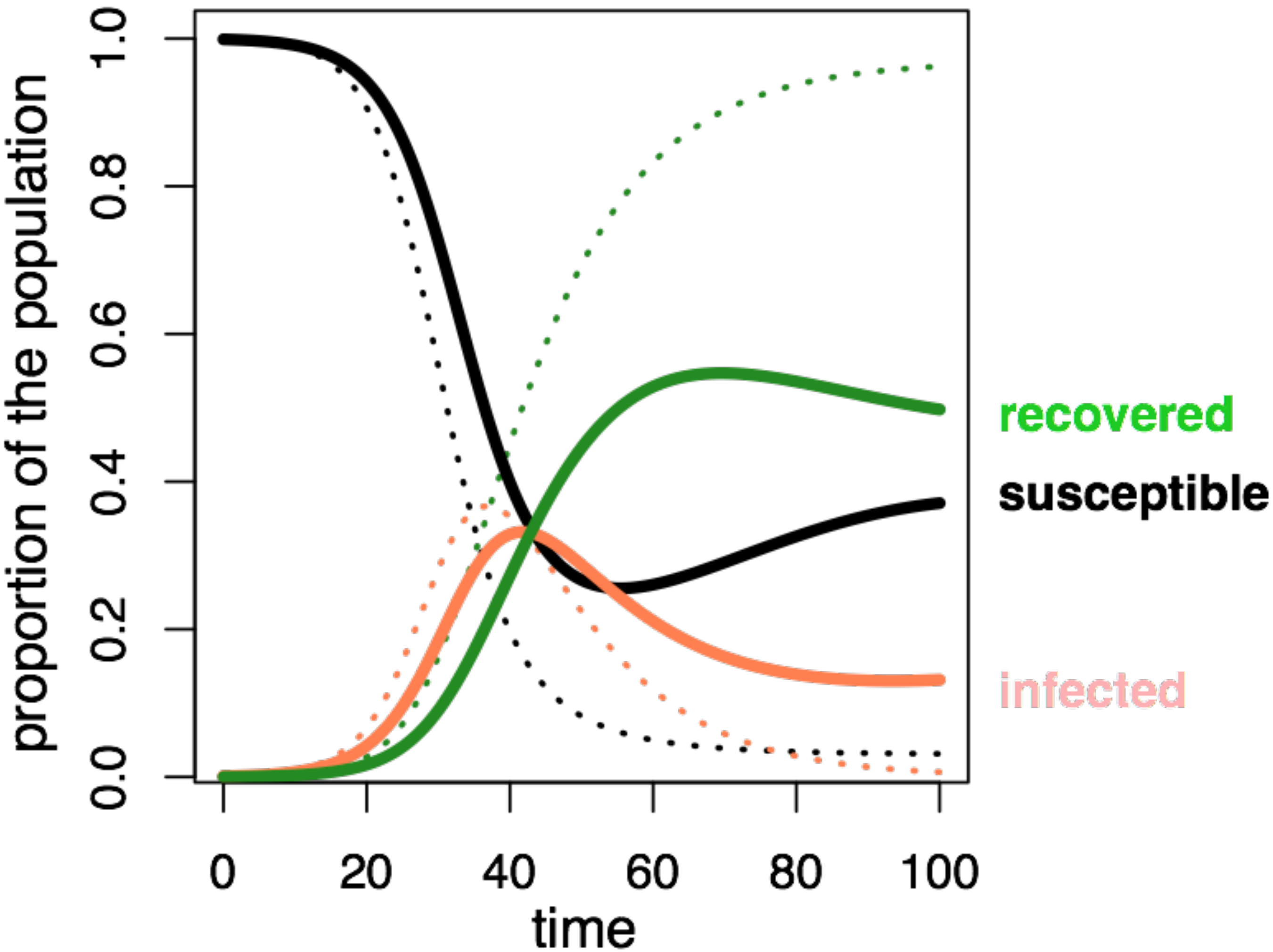
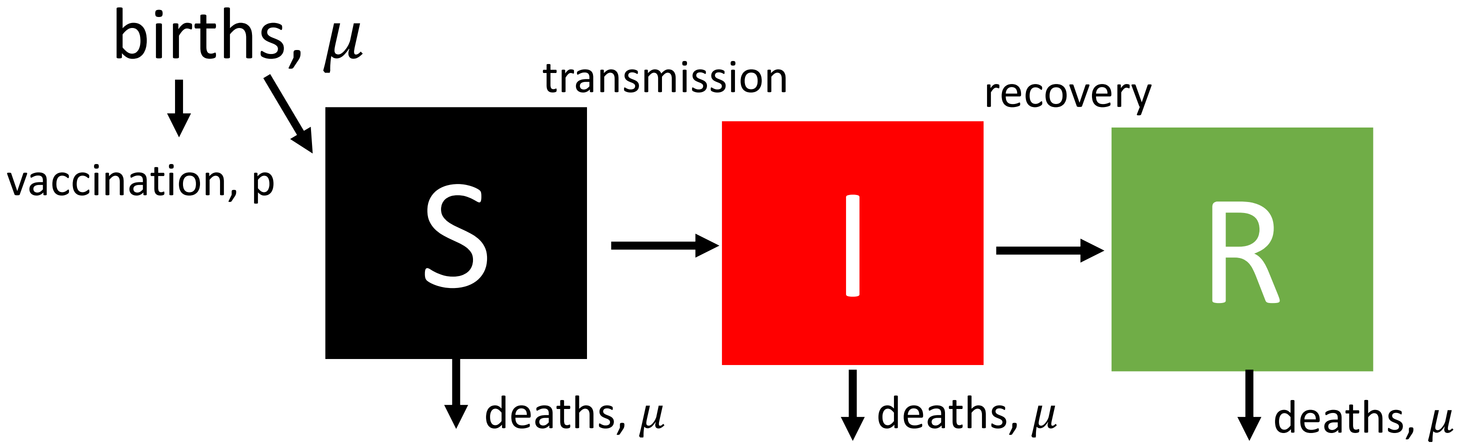
How will births impact dynamics?

The SIR model : add births

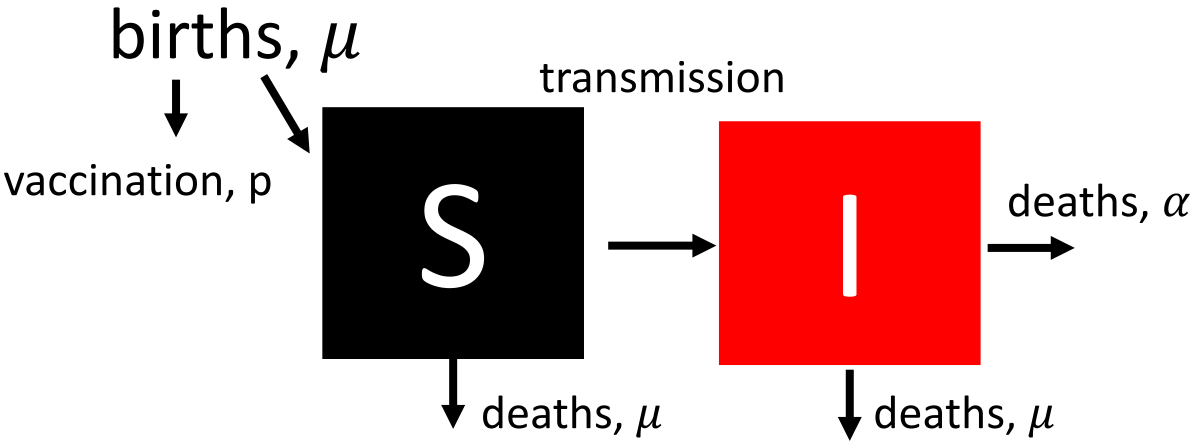


Beyond the SIR model



What do we change if infection is always FATAL?

Beyond the SIR model



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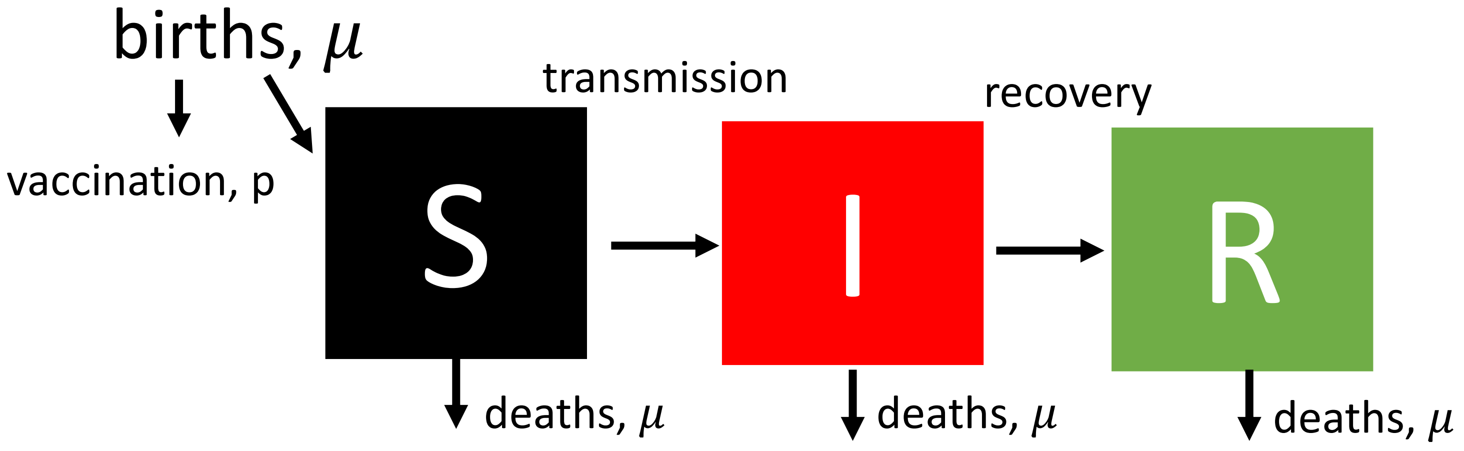
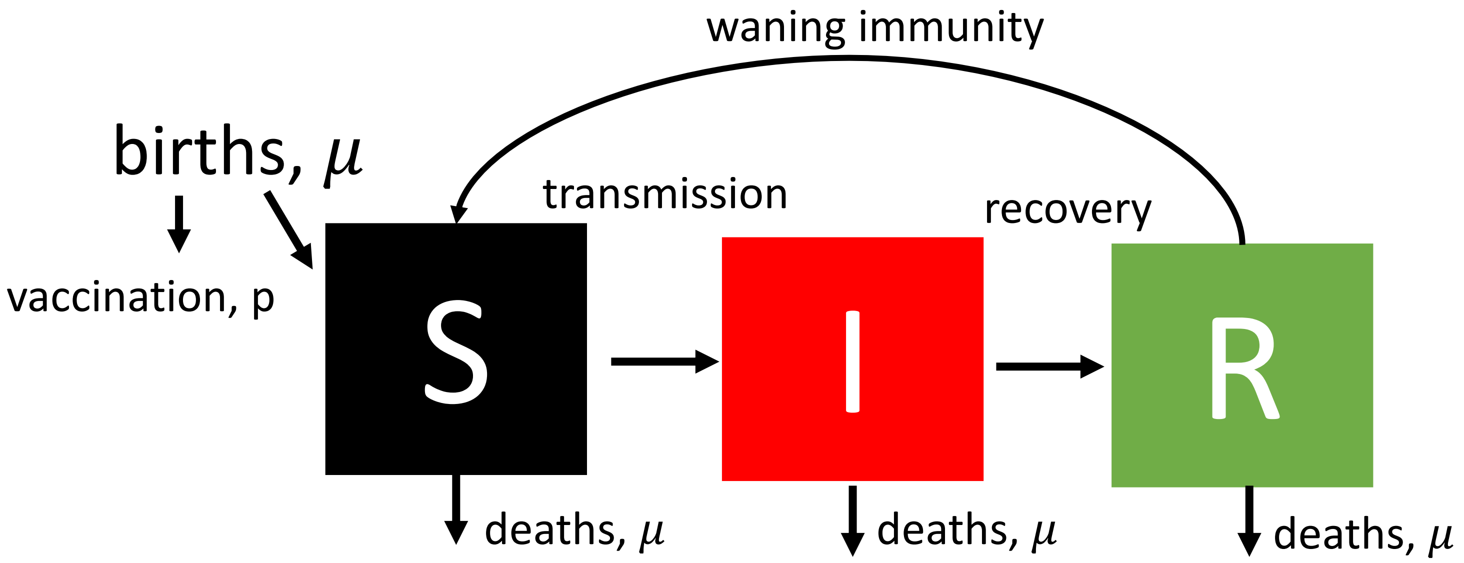


What if immunity wanes?

Beyond the SIR model



What if immunity wanes?

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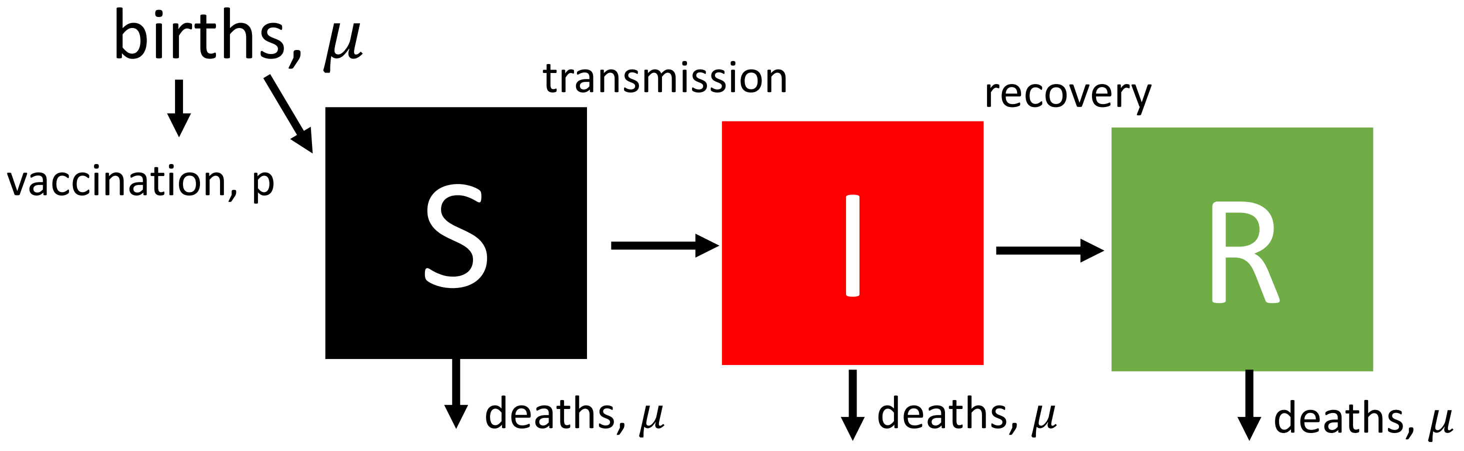
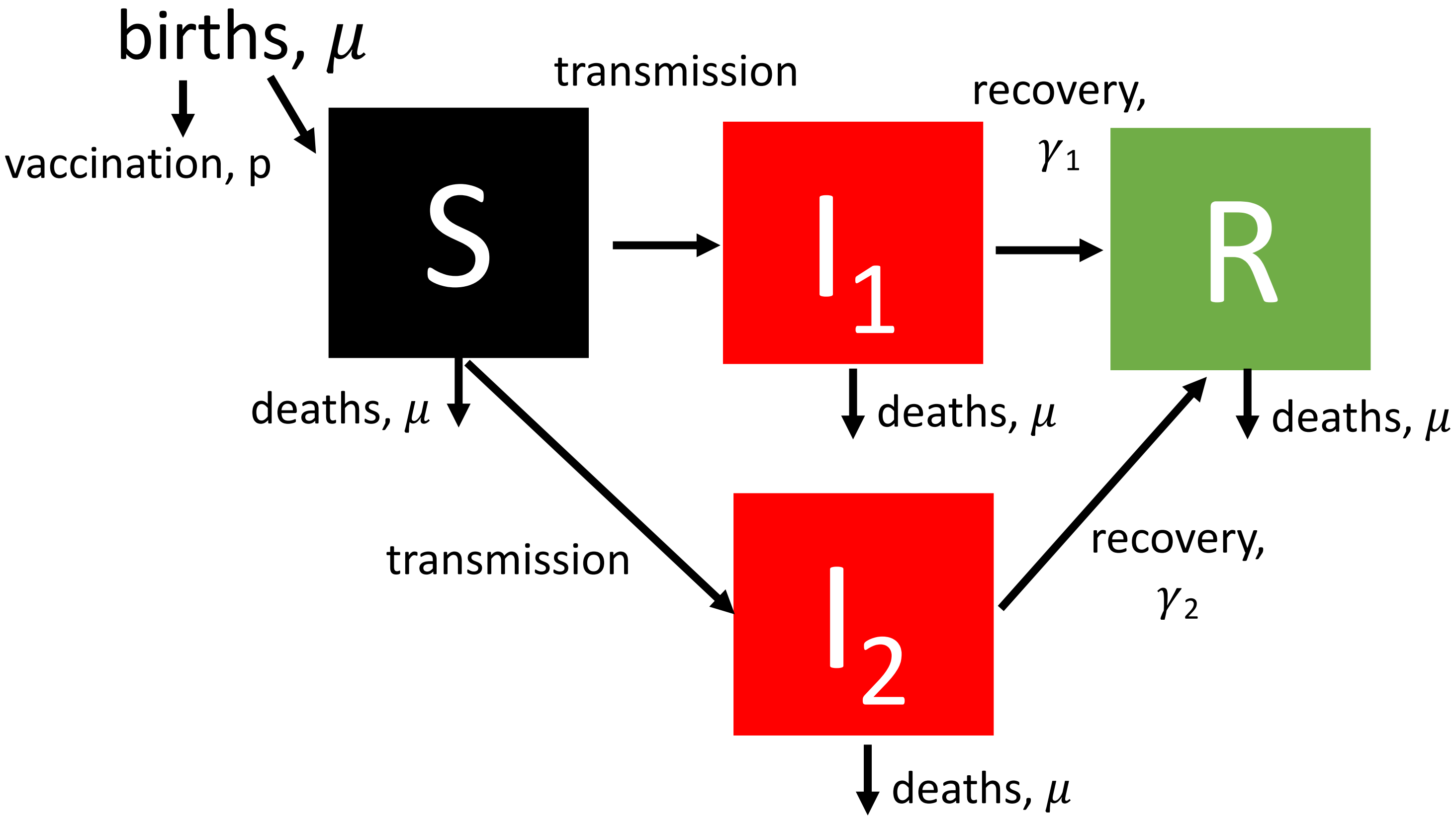


What if people recover at different rates?

Beyond the SIR model



What if people recover at different rates?

Key concepts

- Compartmental/mechanistic/mathematical models

  Modèles en compartiments

- Continuous vs. discrete models

  Modèles en temps continue vs. modèles en temps discrète

- Deterministic vs. stochastic models

  Modèles détérministique vs. stochastique

- Structured models

  Modèles structurés.

- Two population models

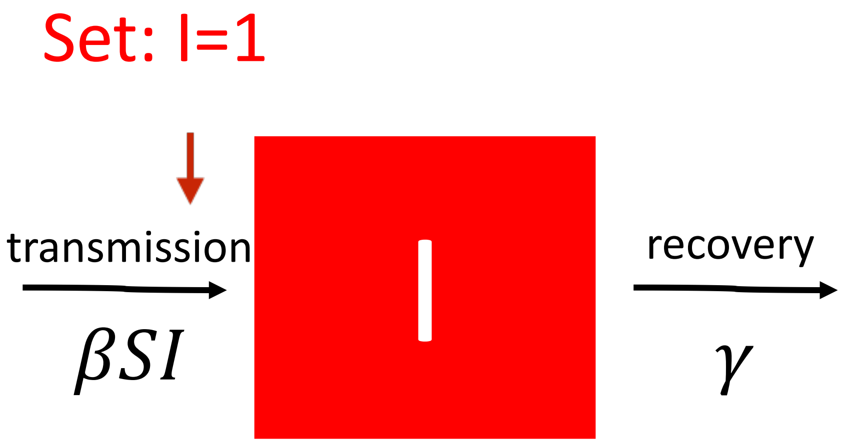
  Modèles des deux populations

- SIR models – and beyond!

  Modèles SIR – et au délà!

"All models are ______, some models are______."

The SIR model



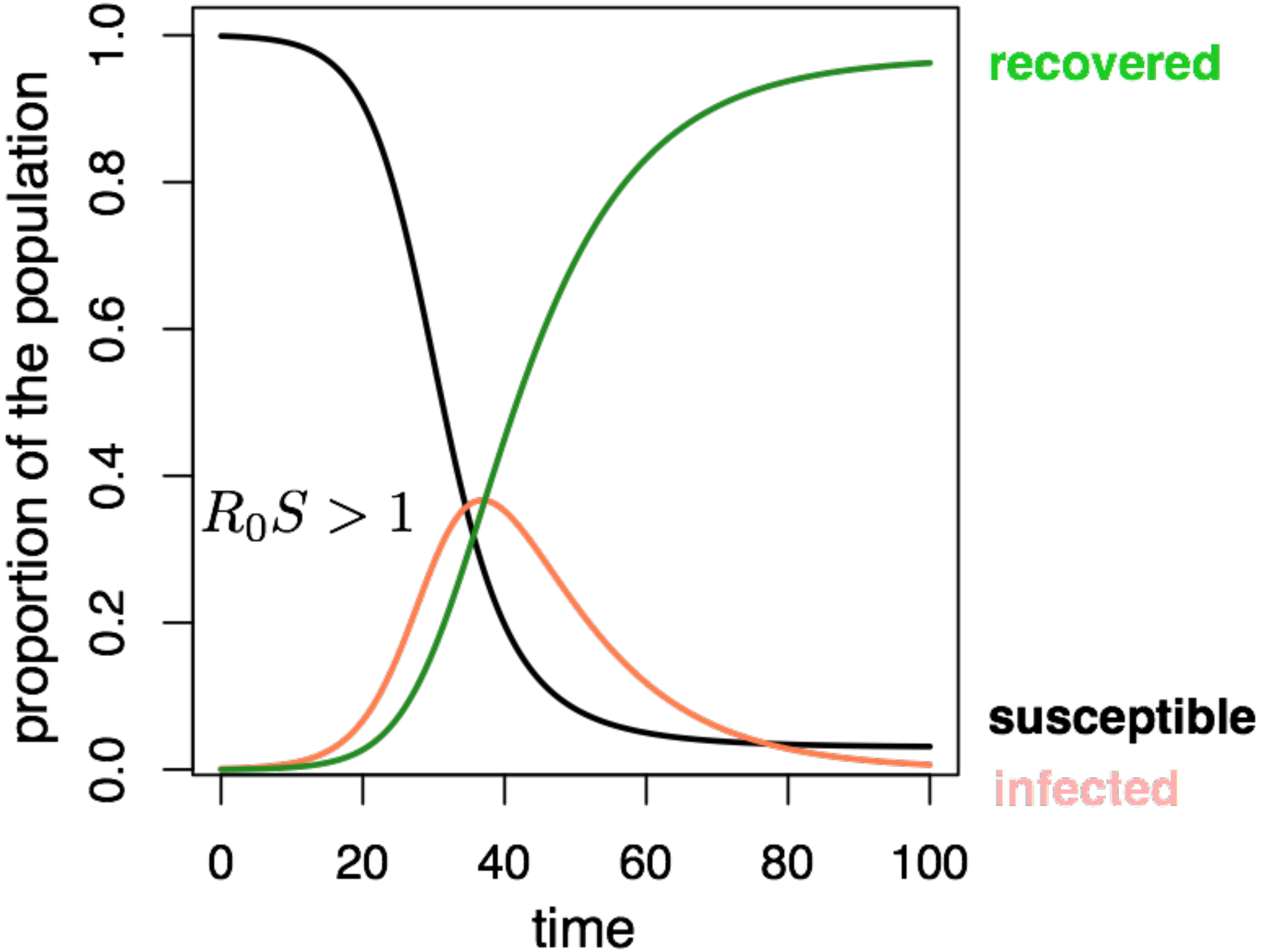
$$R_0 = \beta N/\gamma$$

The average number of persons infected by an infectious individual when everyone is susceptible (S=100%, or S=1, start of an epidemic)

$$R_E = R_0 S$$ “R-effective"

…as the epidemic progresses and S falls

The SIR model



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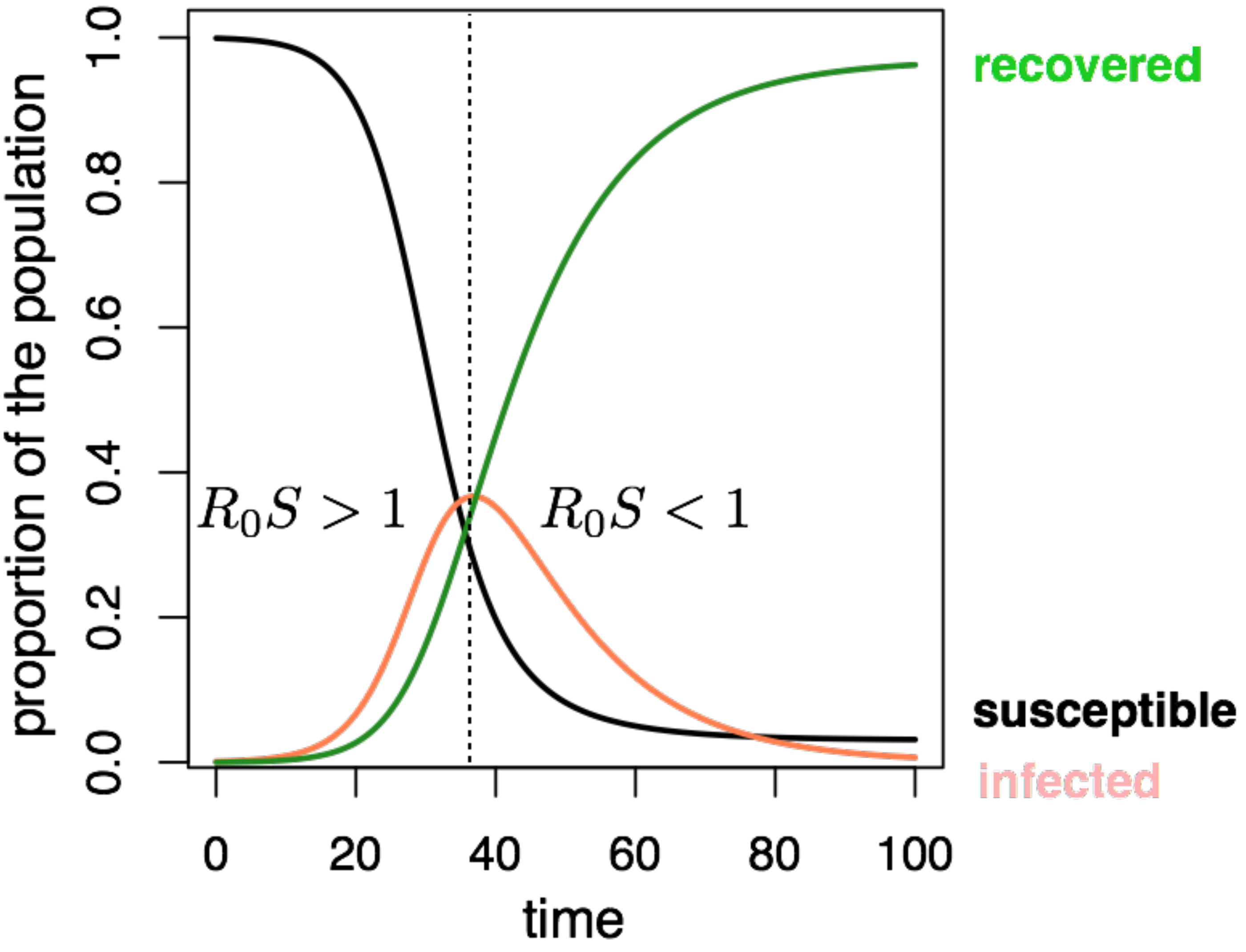
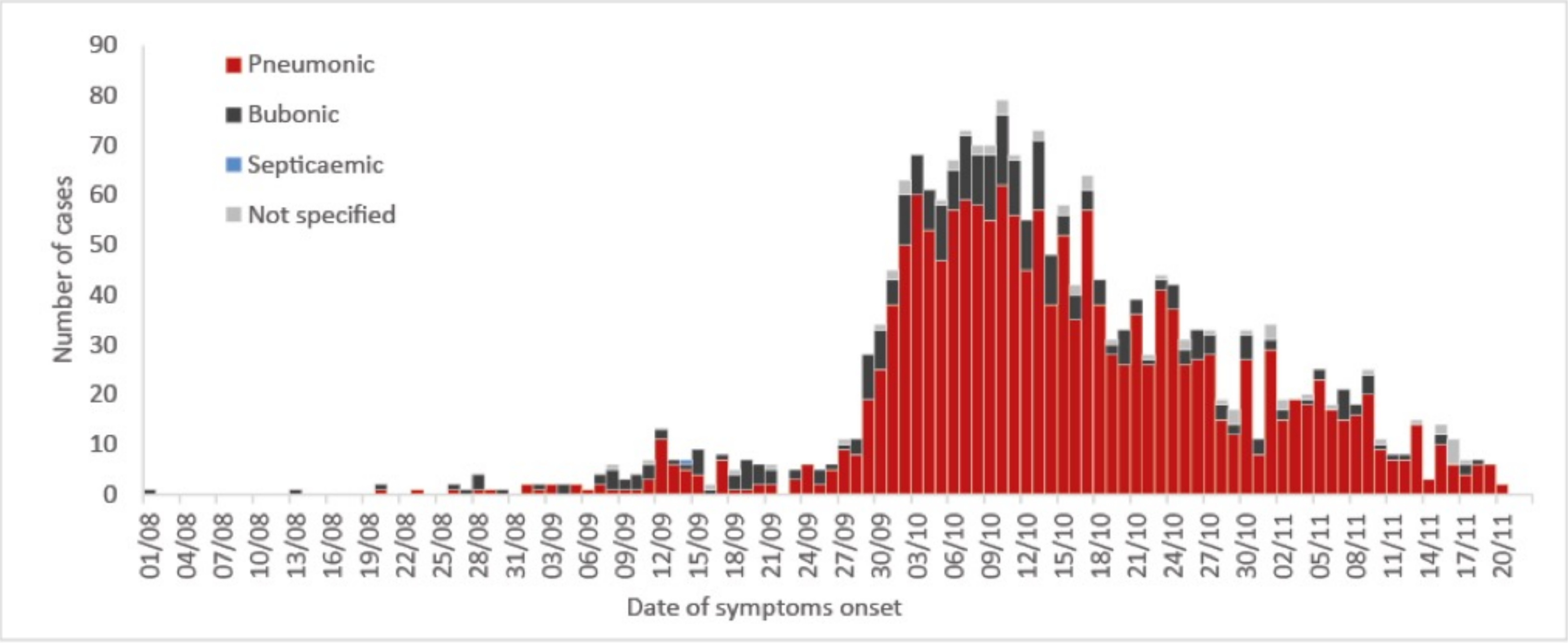


Which model?



http://www.who.int/csr/don/27-november-2017-plague-madagascar/en/