Department of
Ecology and Evolutionary Biology

OPR
OFFICE OF POPULATION RESEARCH

# Model evaluation and comparison

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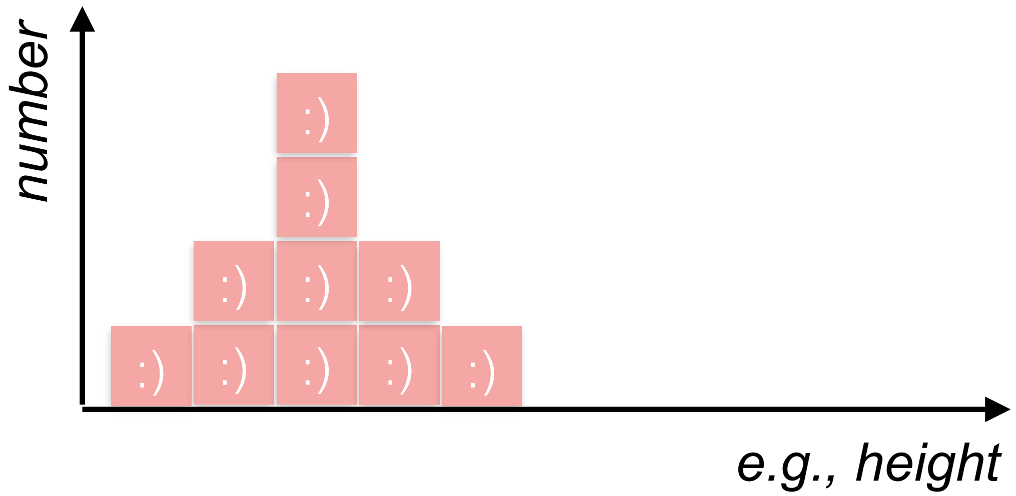
# Data



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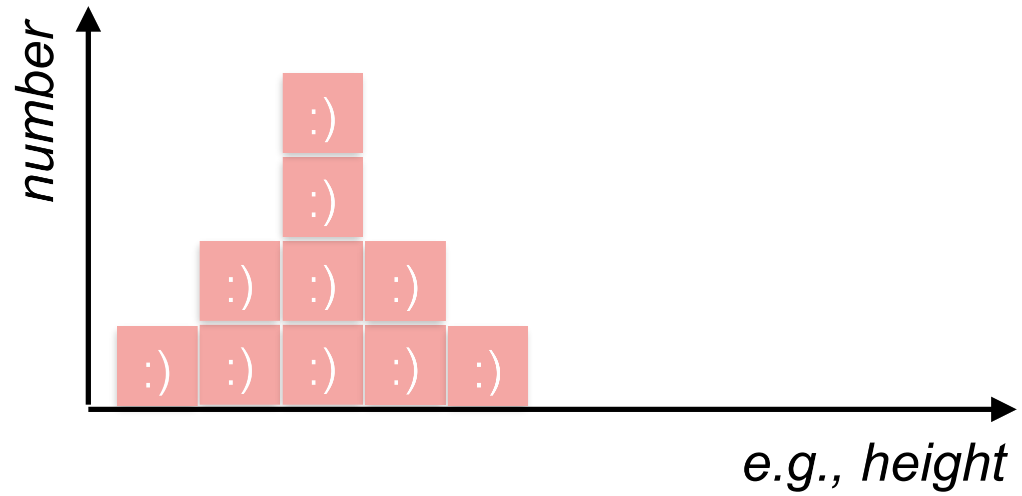
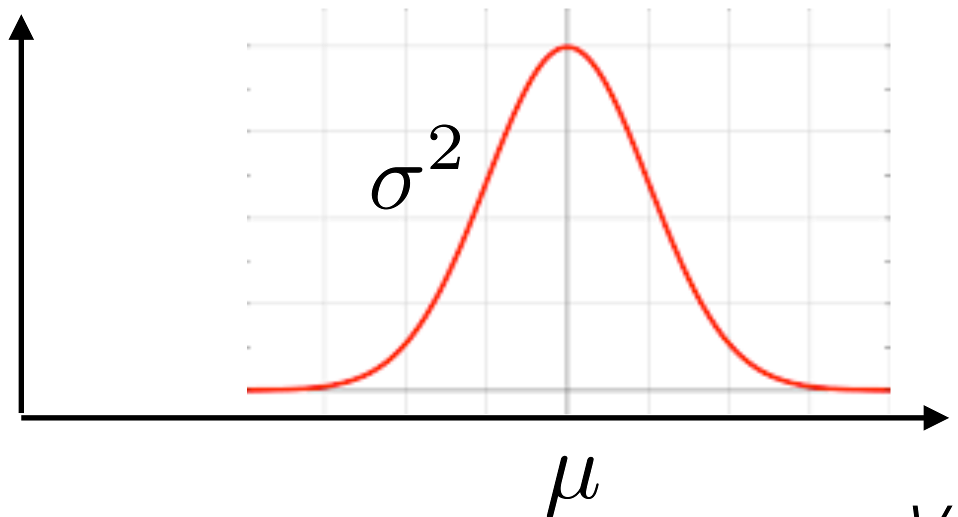


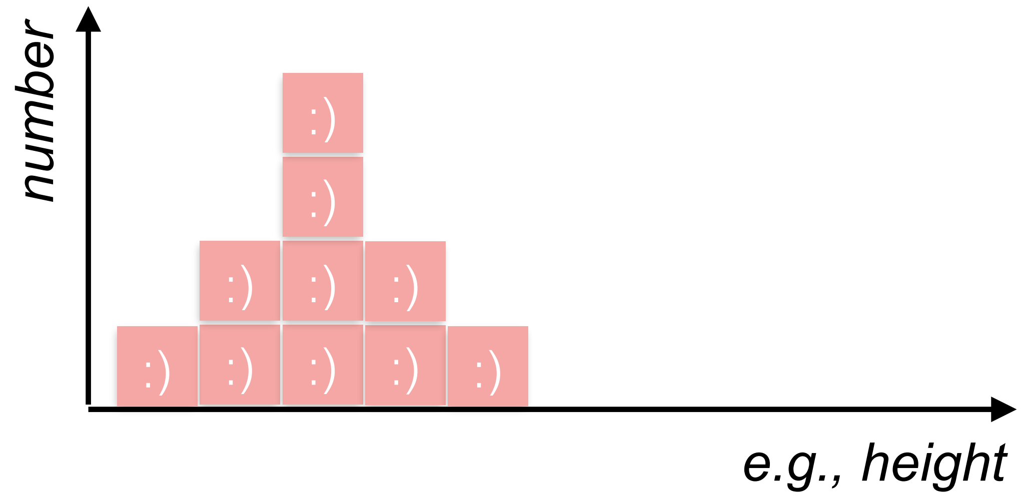

# Likelihood




~ *the* *'probability of seeing the data'* *given the* *chosen parameters*
*…each individual in the data has a corresponding likelihood… multiply them…*

*Very low likelihood.*

# Data

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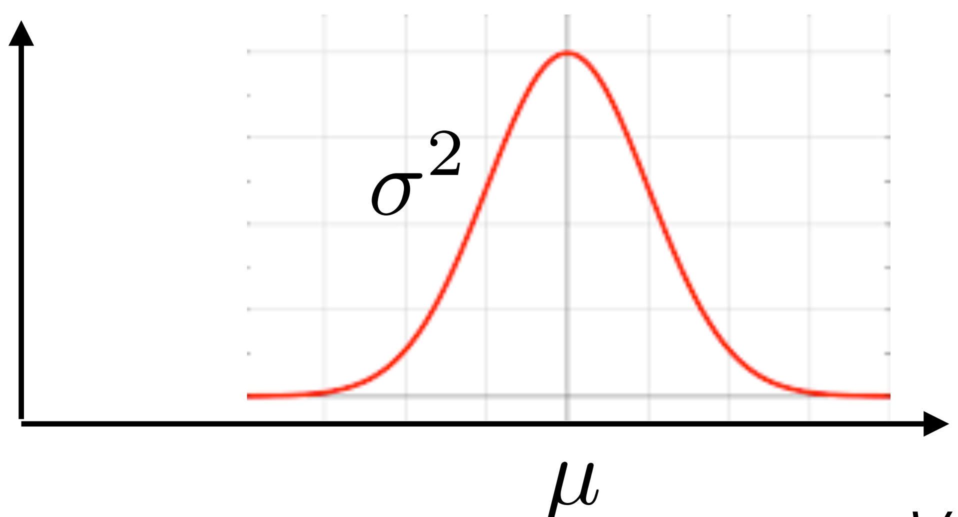
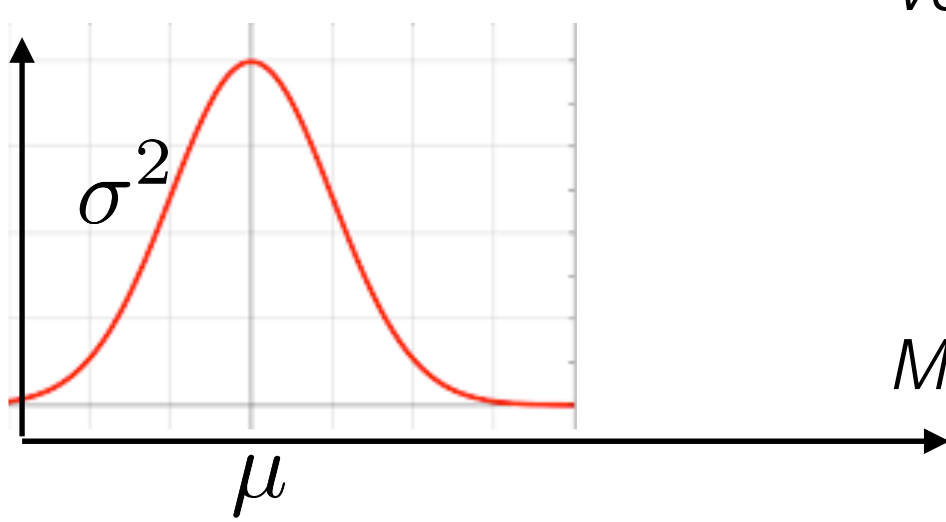



~ the ‘*probability of seeing the data*’ given the *chosen parameters* …each individual in the data has a corresponding likelihood… multiply them…

*Very low likelihood.*



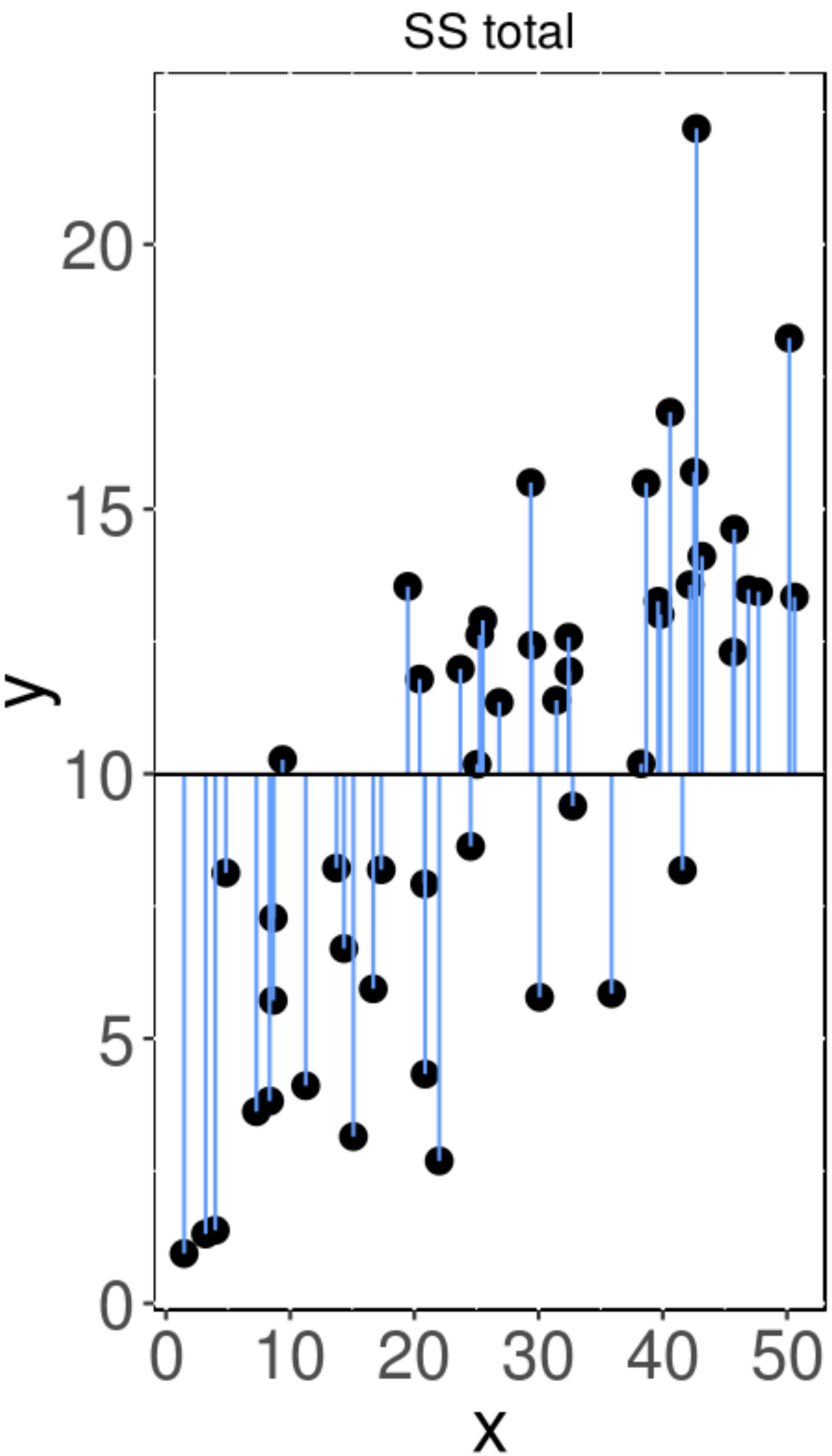

*Maximum likelihood*

What **$r^2$** tells us

**Parsimony,** under- and over-fitting, and AIC

**Sensitivity analysis**

# Definition $r^2$



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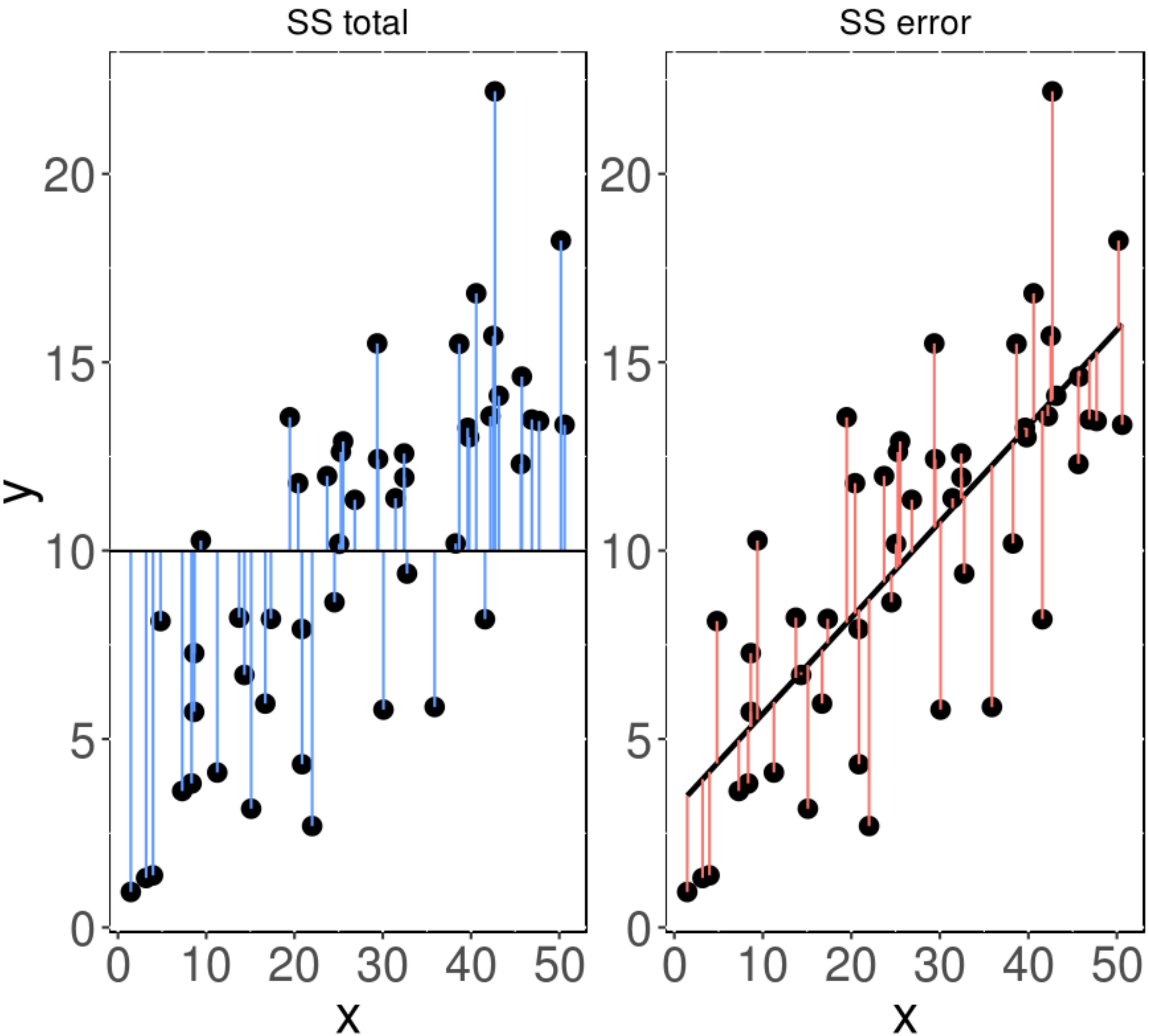

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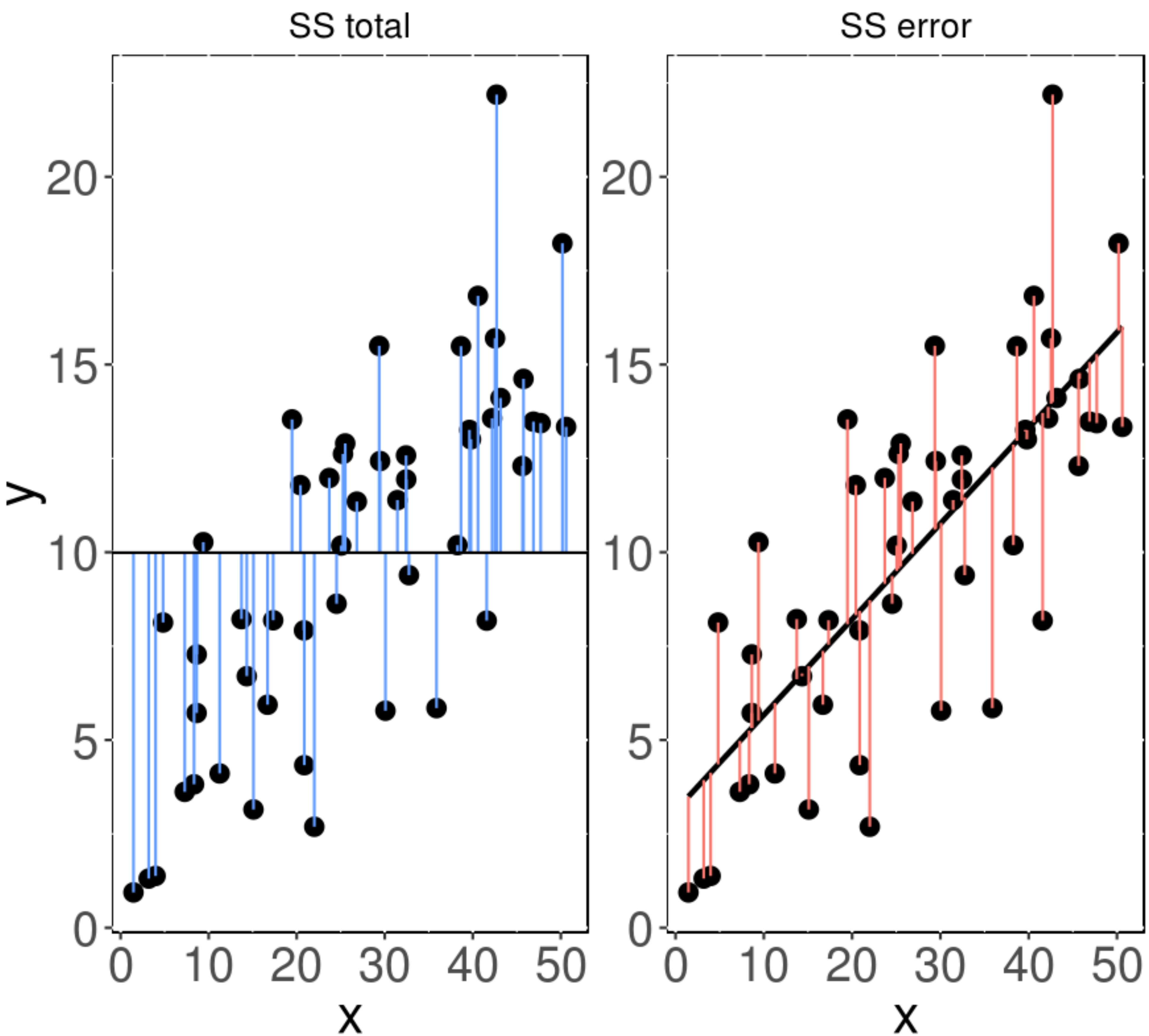
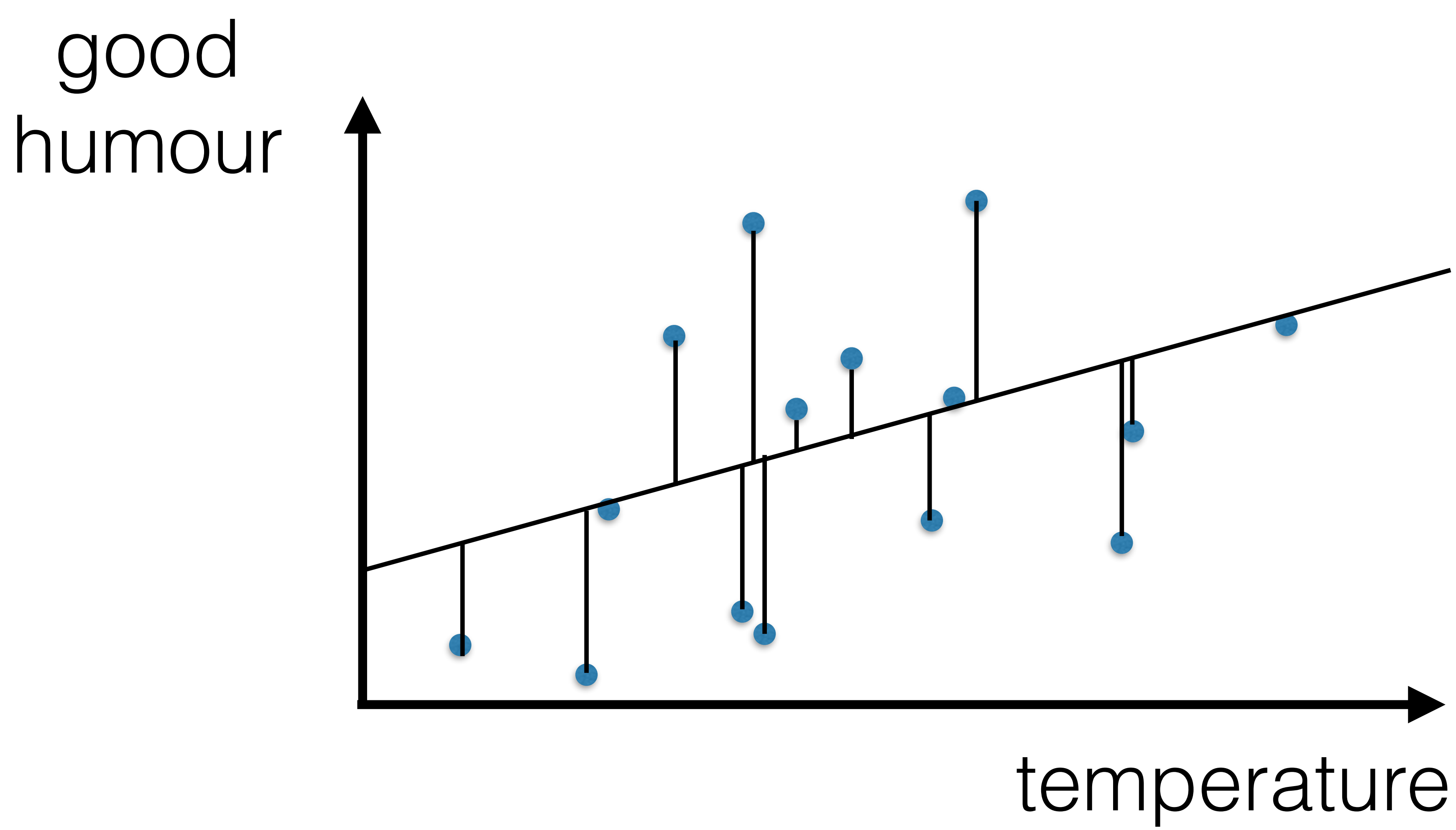



$$R^2 = 1 - \frac{SSE_p}{SST}$$

# Adding covariates and $R^2$




humour = $b_0$ + $b_1$temperature +Error

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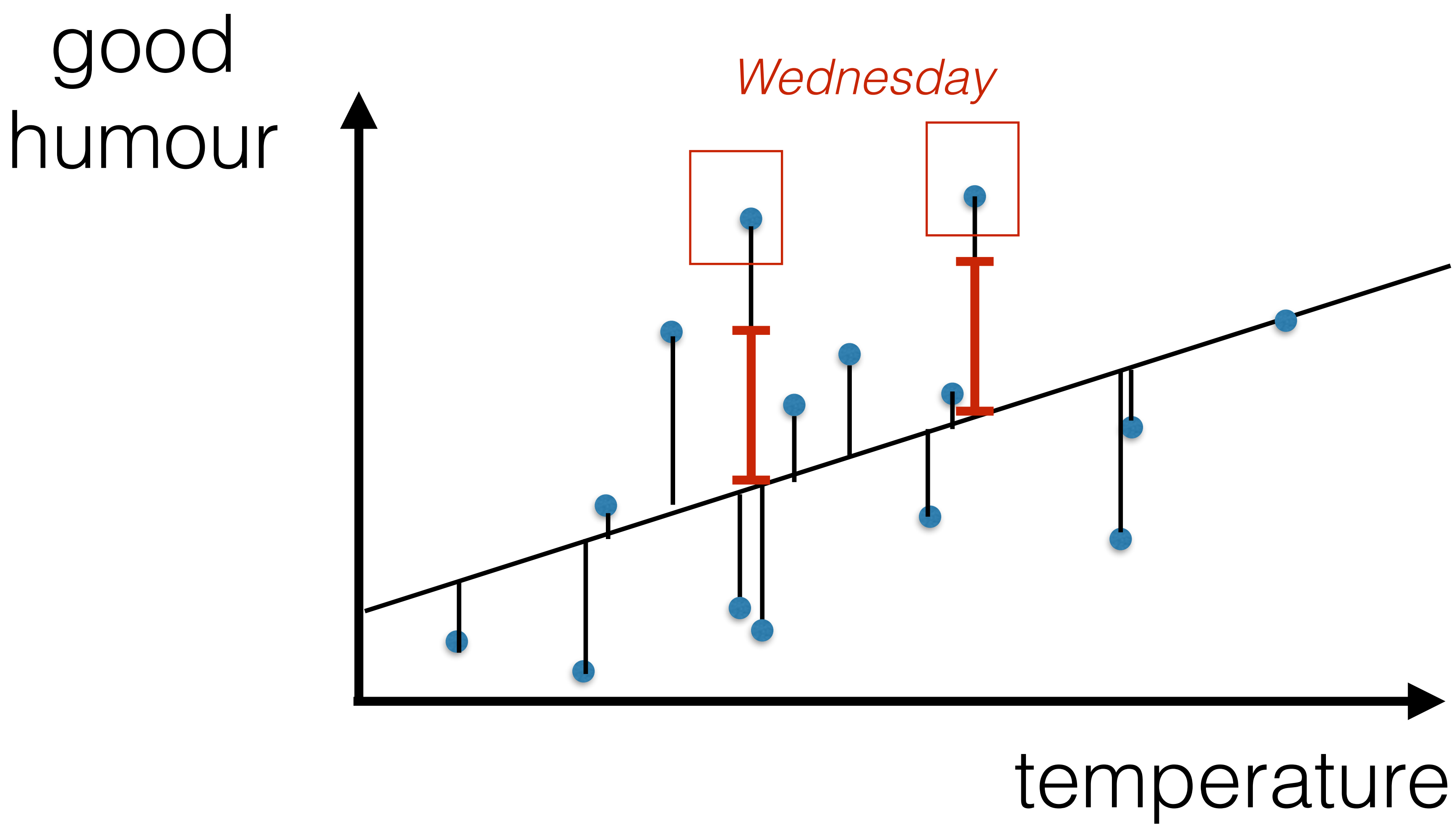
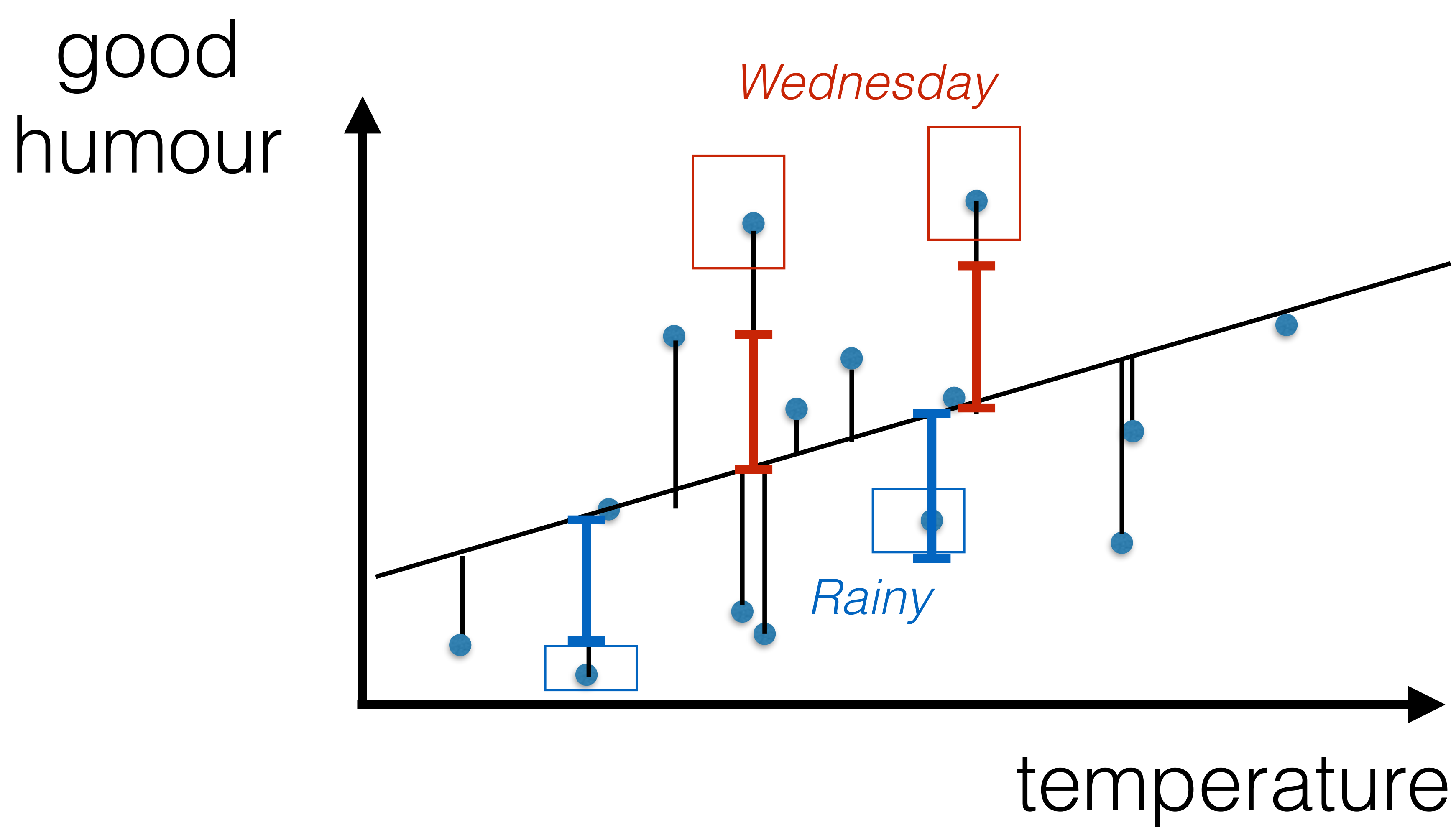


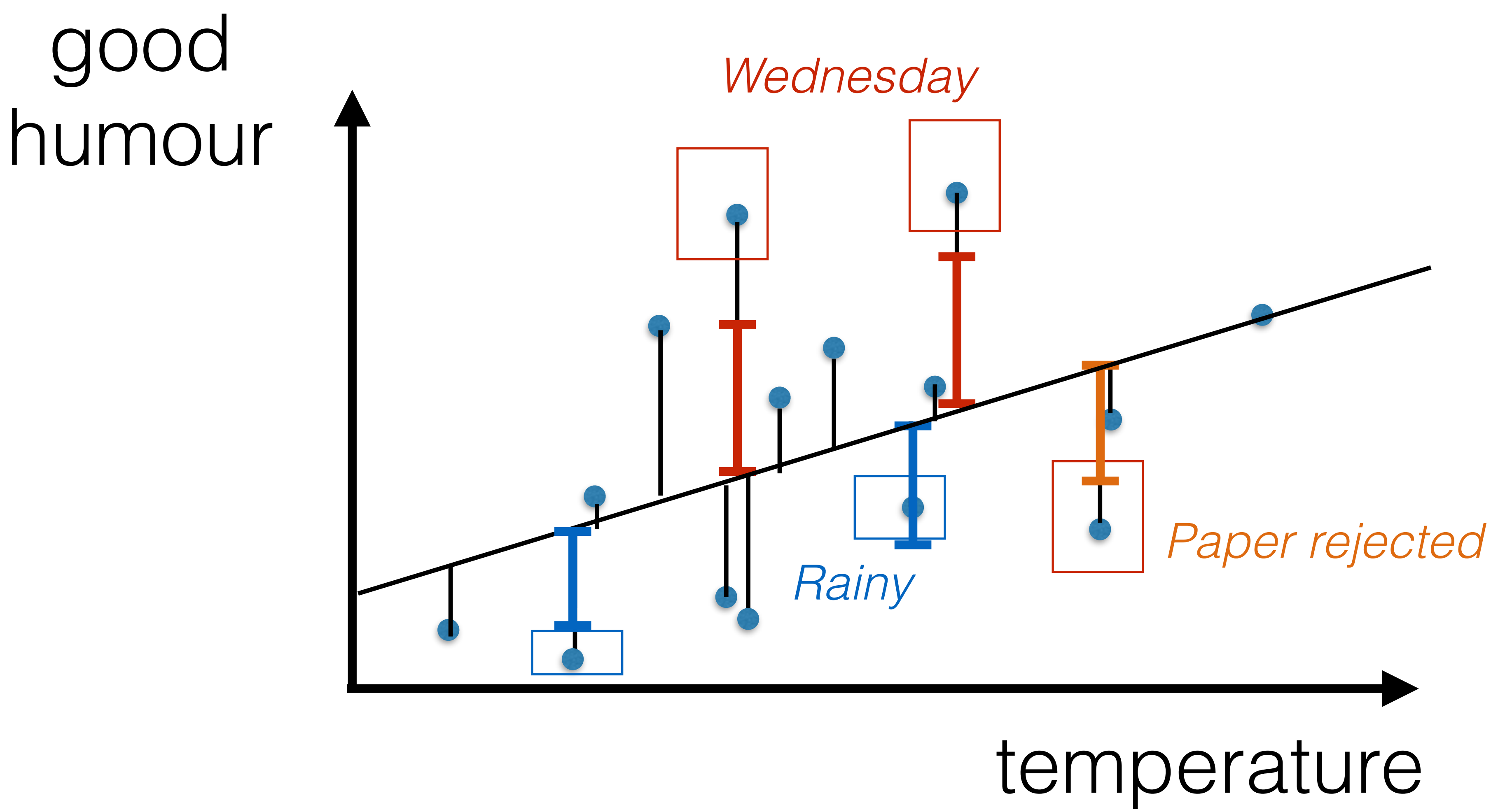

humour = $b_0$ + $b_1$temperature + $b_2$Wednesday +Error

# Adding covariates and $R^2$




humour = $b_0$ + $b_1$temperature + $b_2$Wednesday+ $b_3$rain +Error

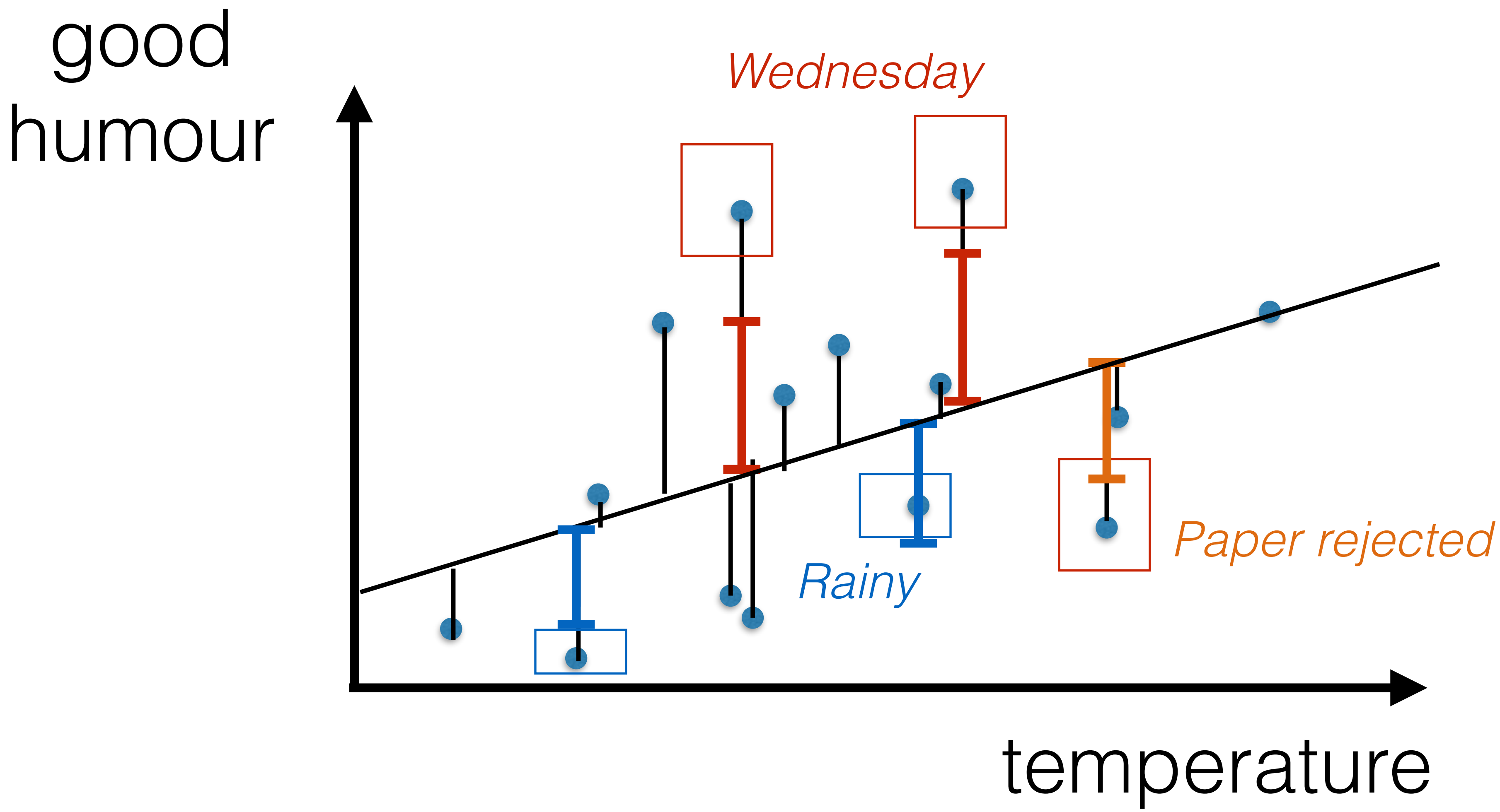
# Adding covariates and $R^2$




humour = $b_0$ + $b_1$temperature + $b_2$Wednesday+

$b_3$rain + $b_4$rejection +Error

# Adding covariates and $R^2$




Adding covariates almost always increases the $R^2$ - so a key question is when to stop.

What **$r^2$** tells us

**Parsimony,** under- and over-fitting, and AIC

**Sensitivity analysis**

# Parsimony, or Occam's razor

William of Occam, 1288-1348

All else being equal, the simplest explanation is the best.

*… model with fewest parameters is the best - as long as it can actually predict reasonably!*

# Interpreting covariates

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

but if $x_1$ is very similar to $x_2$

$$y \sim \alpha + (\beta_1 + \beta_2) x_1 + \epsilon$$

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so?

hard to interpret *mechanistically*
hard to extrapolate across *contexts*

# A good model should be:

- Parsimonious
- Conform to data
- Balance conformity to data and parsimony

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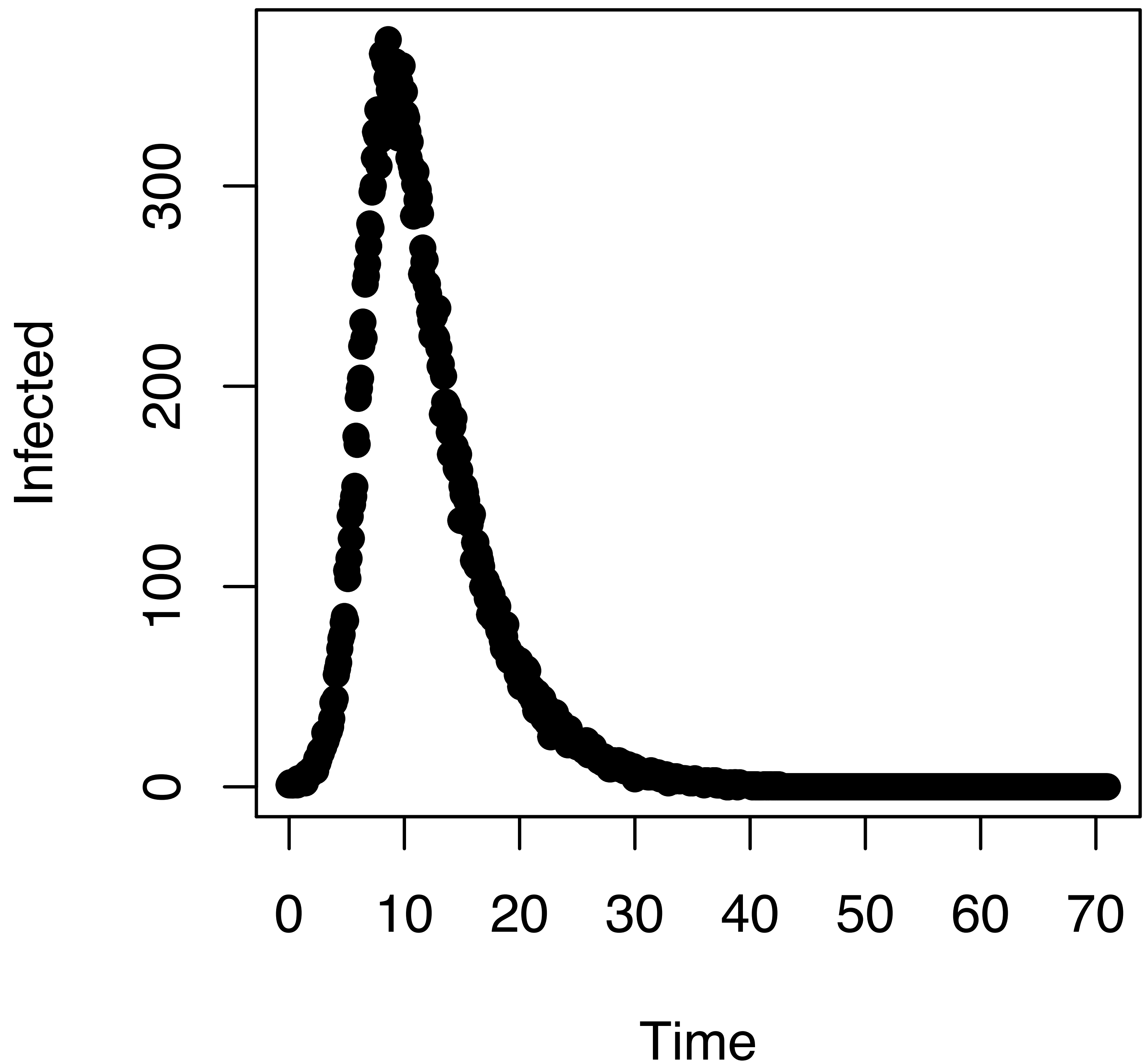
- Easily generalizable

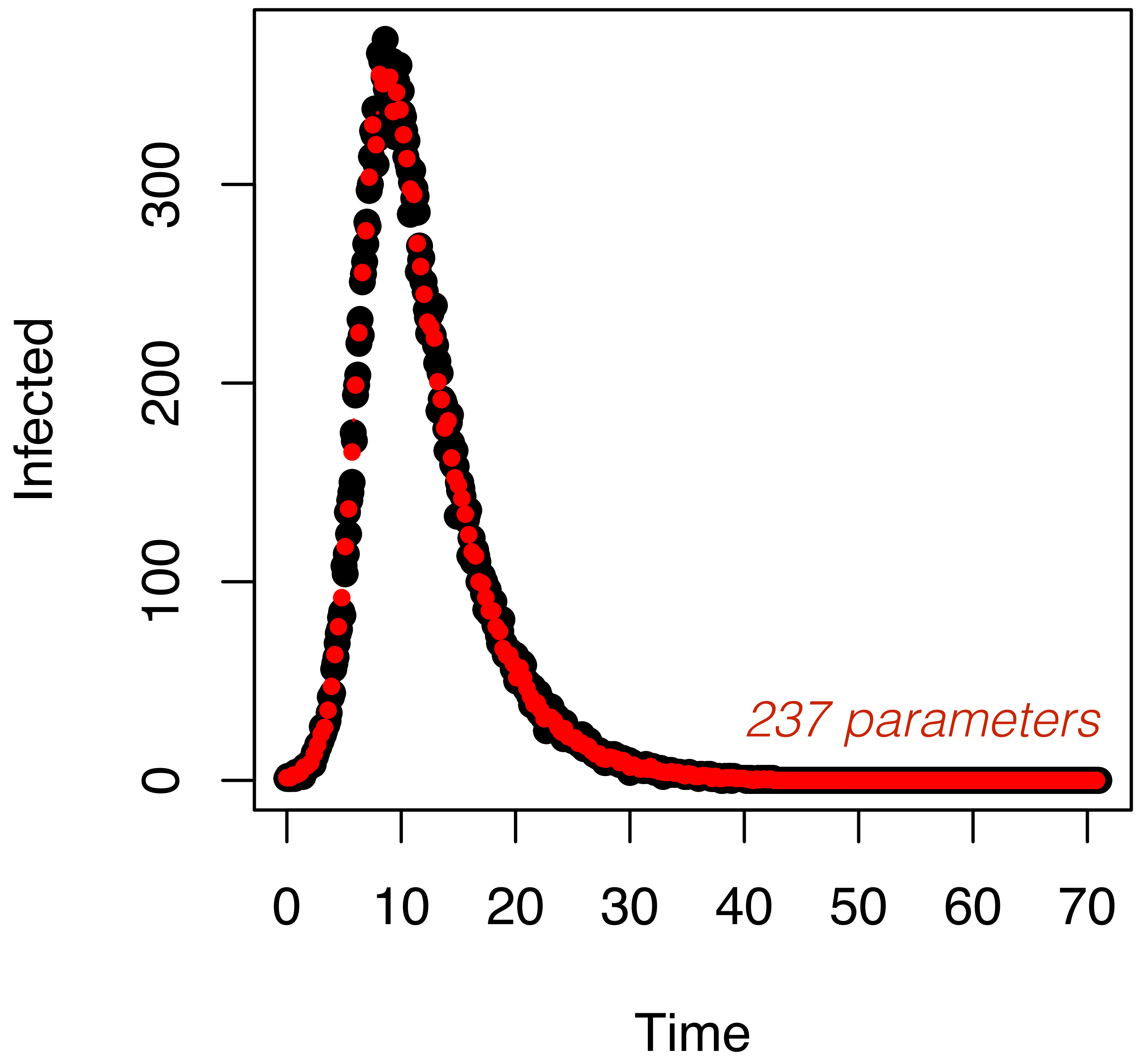
# An outbreak



# Over-fitting

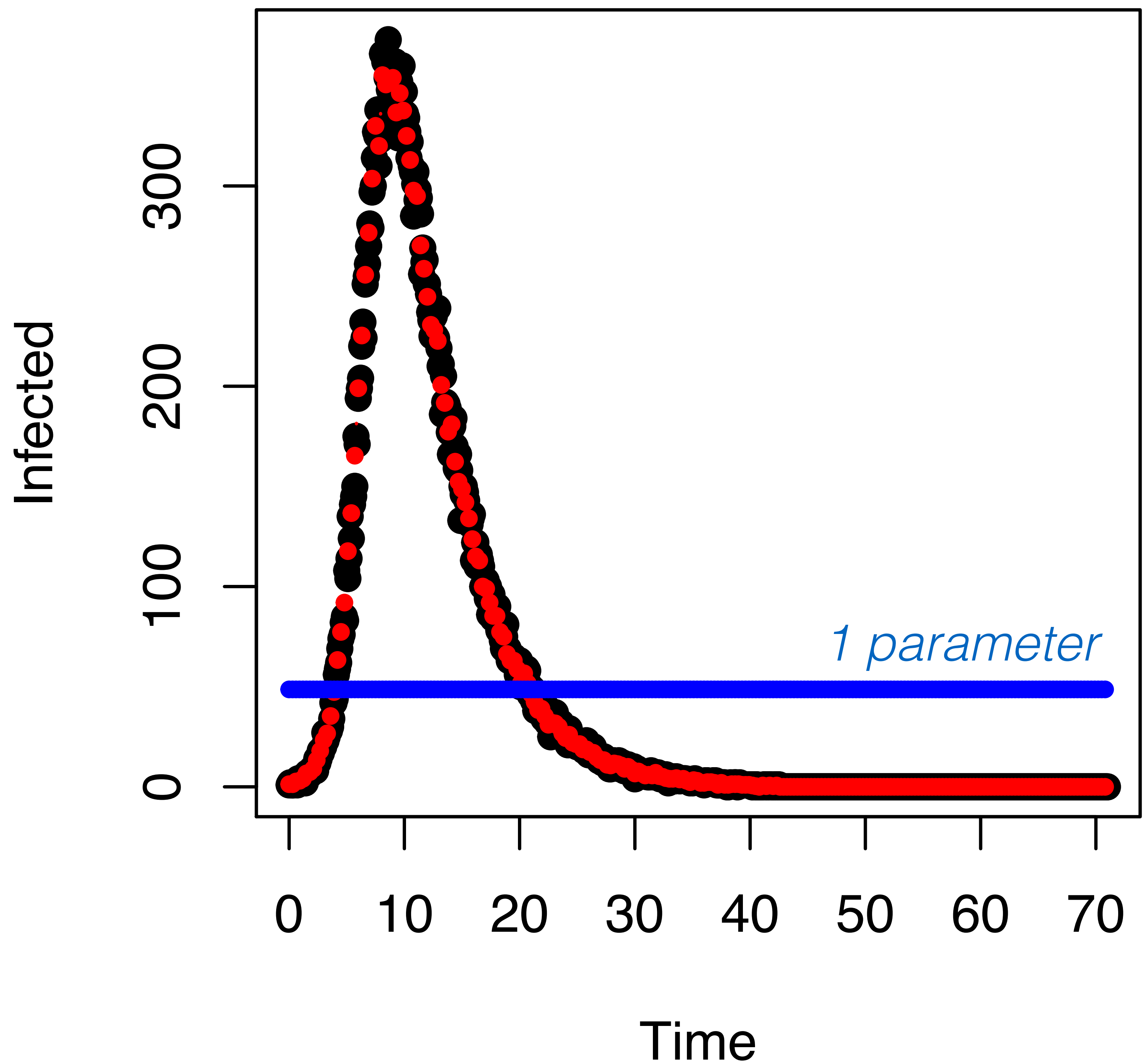
Here, get the mean per three data-points.

In the extremes, one parameter per data-point.



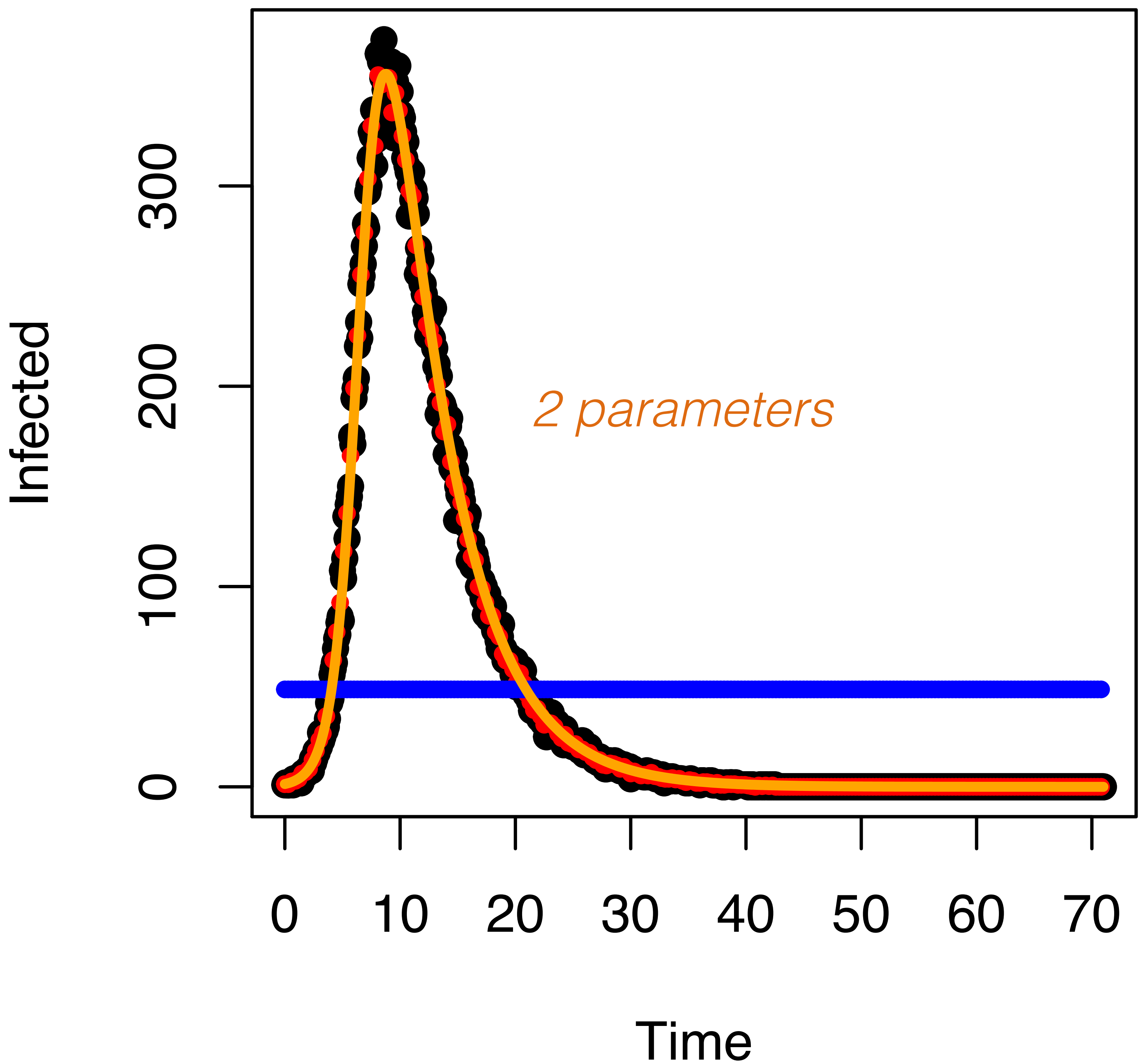
# Under-fitting

Here, one mean for all data points



# Likelihood fitting

Here, a mechanistic (SI) model, i.e., two parameters.




$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$

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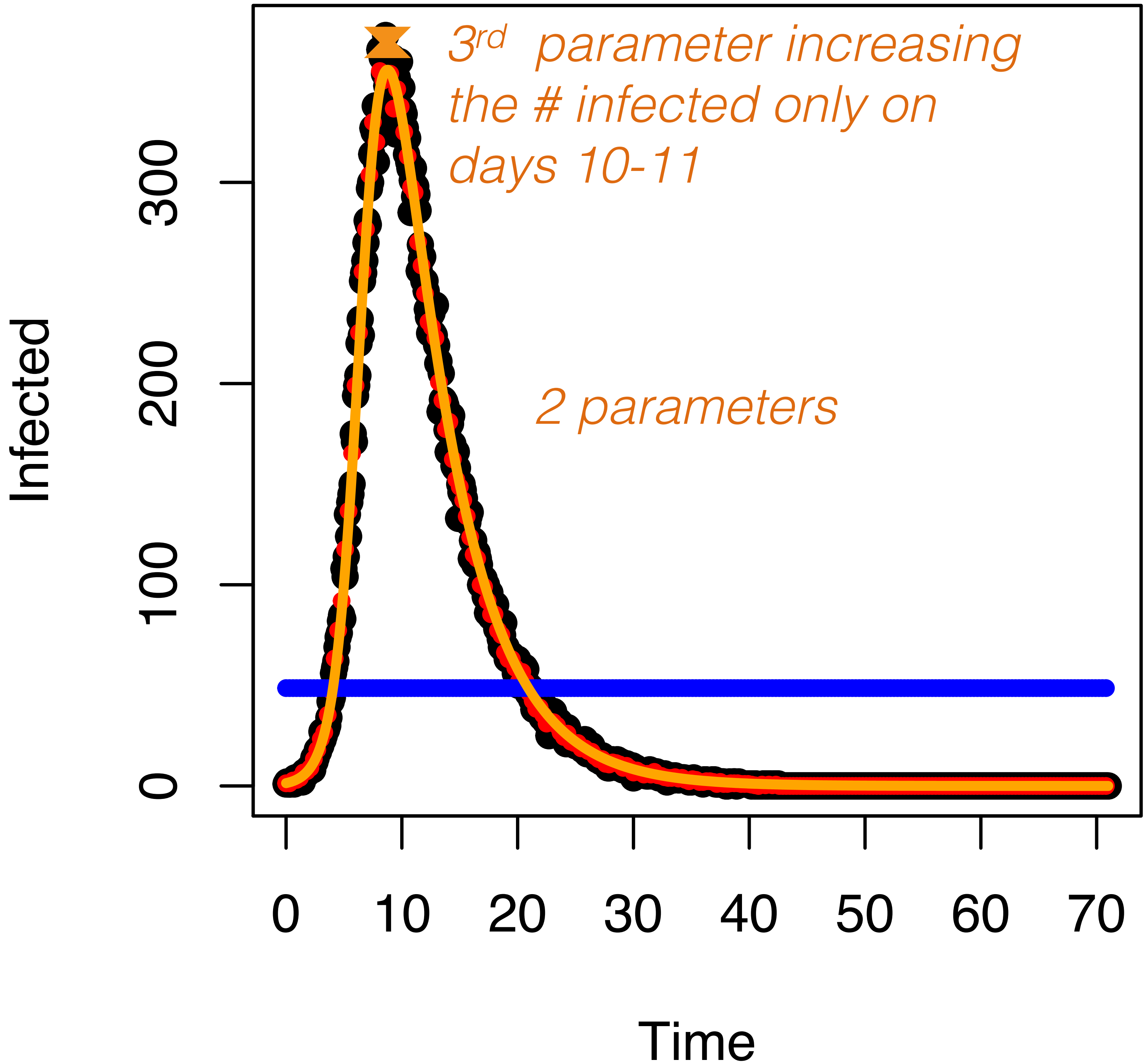


$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$

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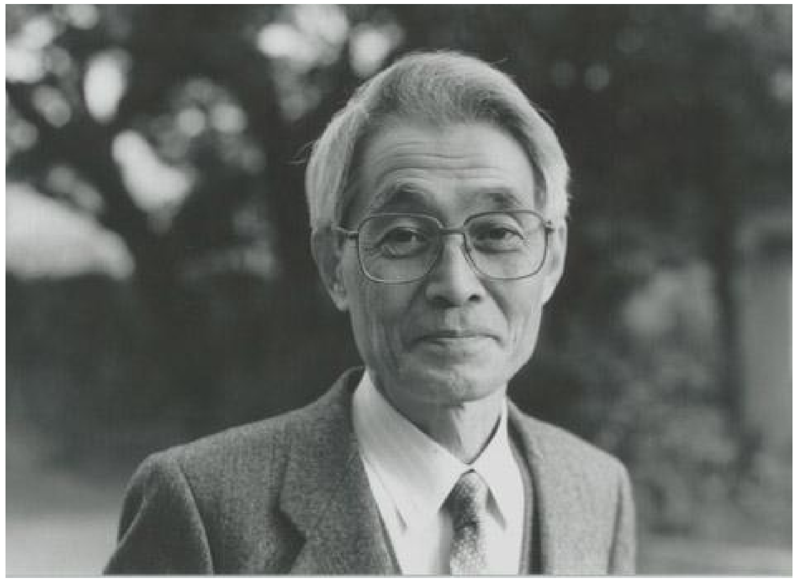
# Akaike’s Information Criteria

Hirotugu Akaike, 1927-2009.



In the 1970s he used **information theory** to build a numerical equivalent of Occam's razor.

AIC = 2K-2logL

K is the number of **parameters** (‘penalty’)
logL is the **log Likelihood** (‘goodness of fit’)

The model with **the lowest AIC is the preferred.**

# Akaike’s Information Criteria

**Advantages:**

Does not require one candidate model to be correct

Can compare nested and un-nested models

Can compare models of different families of probability distributions

**Disadvantages:**

Needs reasonable amounts of data

What **$r^2$** tells us

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**Sensitivity analysis**

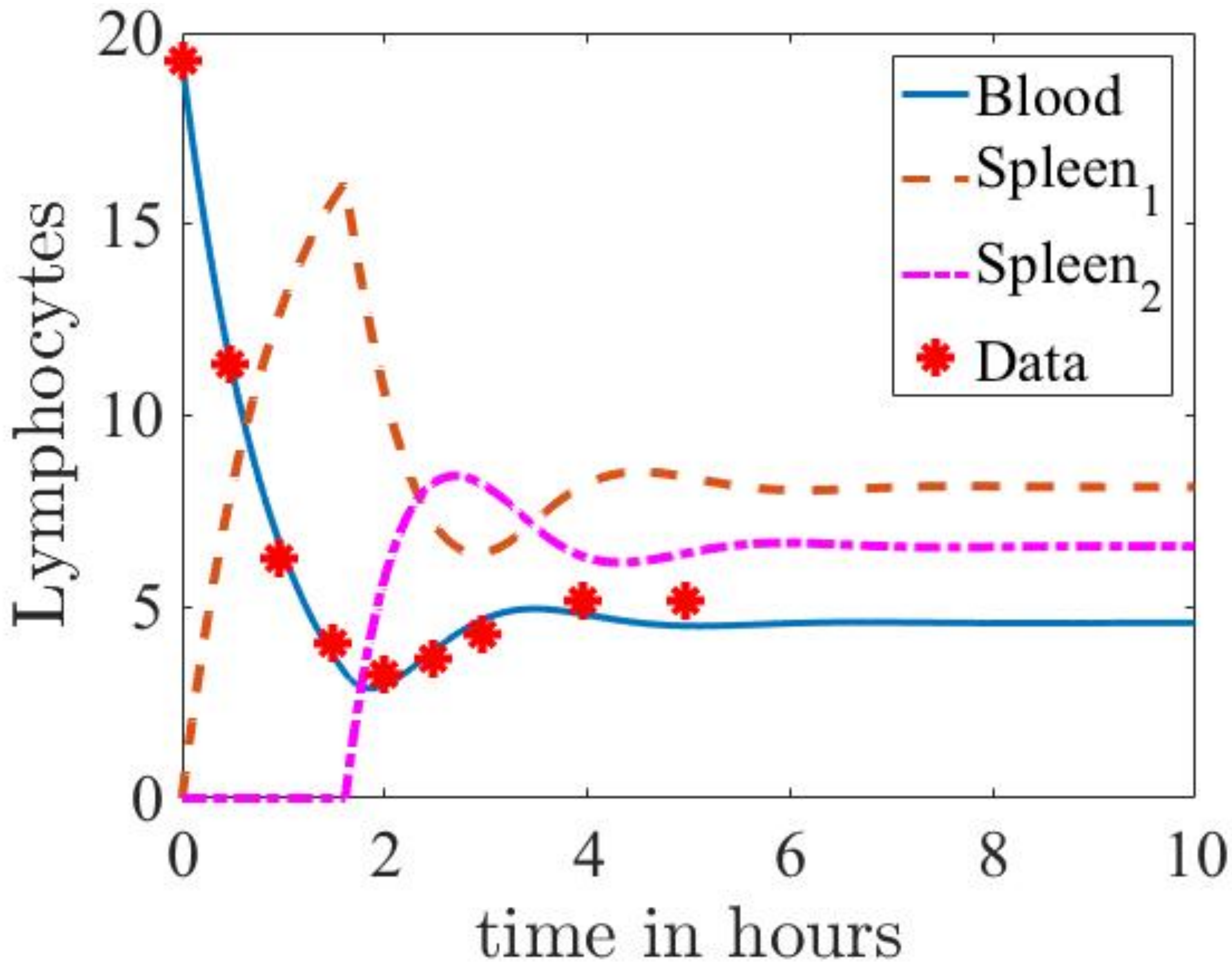
# Sensitivity analysis

- Identifying parameters to which output is **sensitive**
- Identifying **intervention** parameters
- Testing **robustness** of model **predictions**
- Explore **uncertainty** in parameters
- Model **simplification**
- Explore **associations** between parameters

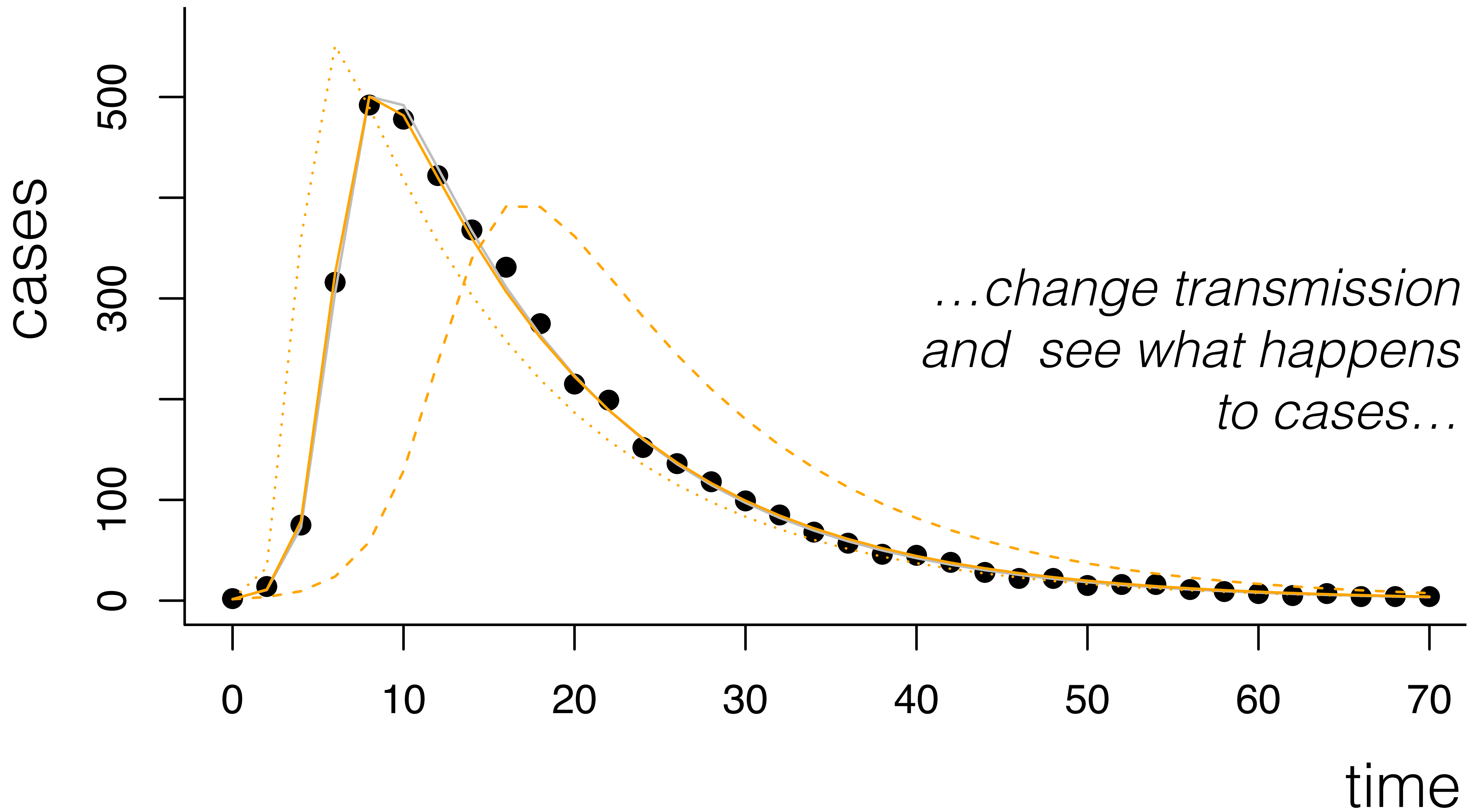
# Example: lymphocyte migration between blood & spleen

$$\frac{dB}{dt} = p_3 S_2 - p_1 B$$
$$\frac{dS_1}{dt} = p_1 \mathrm{B} - p_2 S_1$$
$$\frac{dS_2}{dt} = p_2 S_1 - p_3 S_2$$

$$p_1 = 1.0967$$
$$p_2 = 0.7638$$
$$p_3 = 0.7638$$

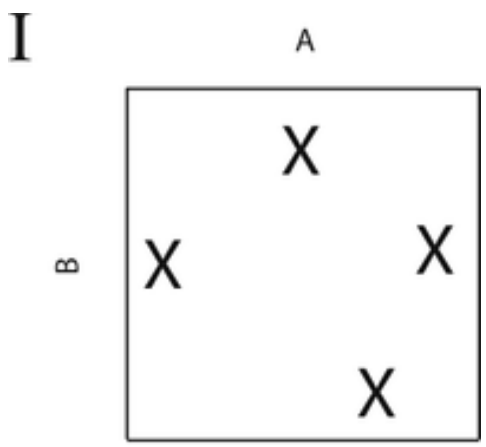


# Example: changing the magnitude of transmission



# Global sensitivity analysis

Evaluate impact of values across the full range possible for all parameters.




Random sampling

# Global sensitivity analysis

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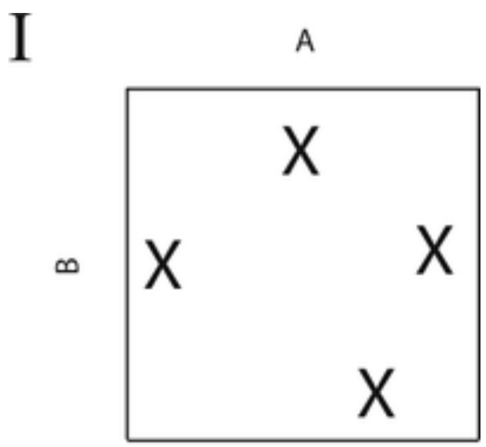
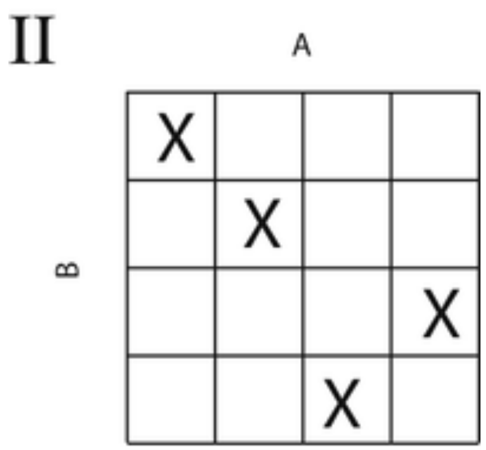



Random sampling




Latin hypercube sampling

Decide on # sample points; then get one sample in each row & column

# Conclusions

A good model should balance **conformity to data** and **parsimony**

**$R^2$** tells us how well models fit ('conformity'), but increases with the number of covariates

Metrics such as **AIC** provide a means to evaluate model parsimony

**Sensitivity analysis** is key to understanding the larger context of parameters (generalizability, etc) and tools exist to do it efficiently (LHS)

# Conclusions

Un bon modèle devrait équilibrer **conformité aux données, et la parcimonie**

**$R^2$** nous décrit la capture des données - mais augmente toujours avec le nombre de covariés

Des metrics tel le **AIC** nous donnes le moyen d'évaluer la parcimonie d'un modèle relatif a d'autres modèles.

L'analyse de la **sensibilité** est clef pour comprendre les paramètres dans un context plus large (si ils décrivent le cas general, etc) et des outils existent pour le faire efficacement.